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Andreev reflection in graphene nanoribbons

Diego Rainis,1,6 Fabio Taddei,1 Fabrizio Dolcini,1 Marco Polini,1 and Rosario Fazio1,2
1NEST-CNR-INFN and Scuola Normale Superiore, I-56126 Pisa, Italy
2International School for Advanced Studies (SISSA), via Beirut 2-4, I-34014 Trieste, Italy

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We study Andreev reflection in graphene nanoribbon/superconductor hybrid junctions. By using a tight-binding approach and the scattering formalism we show that finite-size effects lead to notable differences with respect to the bulk-graphene case. At subgap voltages, conservation of pseudoparity, a quantum number characterizing the ribbon states, yields either a suppression of Andreev reflection when the ribbon has an even number of sites in the transverse direction or perfect Andreev reflection when the ribbon has an odd number of sites. In the former case the suppression of Andreev reflection induces an insulating behavior even when the junction is biased; electron conduction can however be restored by applying a gate voltage. Finally, we check that these findings remain valid also in the case of nonideal nanoribbons in which the number of transverse sites varies along the transport direction.

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I. INTRODUCTION

Graphene, a flat monolayer of carbon atoms arranged in a two-dimensional (2D) honeycomb lattice, is a newly realized 2D electron system1 which has attracted a great deal of interest because of the novel physics which it exhibits and because of its potential as a new material for electronic technology. One of the fascinating aspects of this system is that in most cases its potentialities for device applications are intimately related with fundamental aspects of quantum mechanics. Many of the transport properties of graphene which are at the heart of the design of new functional nanostructures originate from conservation laws of certain quantum numbers. For example, the fact that electrostatic barriers in graphene are perfectly transparent to electron scattering with angles close to normal incidence (Klein paradox) is explained2 in terms of the conservation of pseudospin (the sublattice degree of freedom necessary to describe graphene’s non-Bravais honeycomb lattice1).

Early investigations on transport properties of graphene have analyzed current and noise in the presence of normal leads. Recent studies3–9 have pointed out that novel interesting phenomena arise when graphene is interfaced to a superconductor (SC). In the seminal papers by Beenakker and co-workers3 it was shown that the peculiar band structure of graphene gives rise to the appearance of specular Andreev reflection (AR),10 a novel type of AR that is absent in ordinary metal/SC interfaces. These studies paved the way to experimental investigations of the proximity effect11,12 and of supercurrent flow13,14 in graphene. On the theoretical side, the results of Ref. 3 have been extended in a number of ways, e.g., to graphene bilayers4 and to the case of interfaces with d-wave SCs,7 whereas further studies have focused on the subgap structure of SC-graphene-SC junctions,6 on crossed AR,7 and on magnetotransport.8

Most of the theoretical analyses carried out so far describe the graphene sheet as an infinite (or semi-infinite) 2D plane and identify two energy scales relevant for transport, namely, the superconducting gap \( \Delta_0 \) and the potential difference between the normal side and the superconducting side, which allows, for instance, to switch from the regime of ordinary AR to the one of specular AR. In graphene ribbons,9 however, the finite size of the sample yields an additional energy scale, \( \delta \), characterizing the mean energy spacing between the ribbon bands. Since the typical ribbon size varies from 10 nm up to 1 \( \mu \)m,15 \( \delta \) can range from 300 meV down to 3 meV, and it is thus larger than (or of the same order of) the typical superconducting gap \( \Delta_0 \lesssim 1 \) meV. As a consequence, \( \delta \) is expected to play an important role in electron transport and AR in graphene.

In the present work we address this problem by analyzing electronic transport through a hybrid junction between a graphene nanoribbon (GNR) (Refs. 16–19) and a SC, as sketched in Fig. 1. We consider the case of a GNR with zigzag edges, which has been shown to represent fairly well the behavior of an arbitrarily shaped edge.20

Remarkable effects emerge due to the finite size of the GNR. We find that AR is strongly affected by the conservation of pseudoparity, a quantum number characterizing the GNR eigenstates, which depends on whether the number of sites along the transverse direction is even or odd. In particular, while in GNRs with odd number of sites along the transverse direction AR is totally suppressed. In the latter case AR can be restored to a finite value by applying a gate voltage. Even-odd effects with the same physical origin have been also found in normal transport through graphene p-n junctions (valley-valve effect).21–23

This paper is organized as follows. In Sec. II we introduce the tight-binding Hamiltonian that we use to model the GNR

![FIG. 1. (Color online) A hybrid junction between a zigzag graphene nanoribbon and a superconductor.](image-url)
where \( n \) to depend on whether \( n = 2 \). One can show that the zigzag GNR at a cell \( r \) can be factorized in a longitudinal, a transverse, and a spin component. More explicitly, where the number \( p \) is the hopping energy between nearest-neighbor sites, \( \gamma \) is the constant electrostatic energy, which can be controlled by a superconducting electrode, and \( \text{int}[x] \) denotes the integer part of \( x \). The transverse momentum \( k_y \) depends on the longitudinal momentum \( k_x \). Details about such relation as well as the explicit expression for the eigenvalues will be discussed elsewhere. Finally the spin-wave function \( w_{\alpha} \) can be chosen as the \( \hat{z} \)-direction spinors \( w_{\uparrow}(x) = 1, 1 \) and \( w_{\downarrow}(x) = 0, 1 \). Importantly, beside momentum and spin, the eigenstates defined in Eqs. (1)–(3) exhibit an additional quantum number \( \eta = \pm 1 \), which determines whether the transverse cell wave function \( \chi_{\pm}(x) \) is even or odd with respect to the longitudinal axis of the GNR. One can therefore term \( \eta \) as pseudoparity. Note that in contrast to what happens in bulk graphene, states (1)–(3) are not eigenstates of the pseudospin-projection operator \( \sigma \cdot n \) for any unit vector \( n \).

The eigenvalues \( e \) obtained from the tight-binding equations as functions of \( k_x \) describe the band structure of the zigzag GNR, where both bands exhibit dispersionless zero modes (corresponding to states localized at the edges of the GNR) and the other ones are somewhat reminiscent of the bulk-graphene Dirac cones, as illustrated in Fig. 3 for the case \( U = 0 \). The presence of the superconducting electrode is accounted for by the Bogoliubov–de Gennes Hamiltonian.
where \( H \) is the particle Hamiltonian and \( \varepsilon_F \) is the Fermi level of the GNR/SC junction at equilibrium. The SC (s-wave) order parameter, which couples particles and holes, is described by a matrix \( \Delta \) with entries \( \Delta_{ij} = \Delta \delta_{ij} \). Here \( \Delta \) is taken to vary smoothly across the junction between its maximum value \( \Delta_0 \) in the bulk of the SC and zero in the bulk of the GNR.\(^{30}\)

Within the scattering approach\(^{31}\) the zero-temperature differential conductance is given by\(^{32,33}\)

\[
G(V) = \frac{dI}{dV} = \frac{2e^2}{h} [N(V) - R(V) + R_A(V)],
\]

where \( V \) is the bias voltage applied across the junction. In Eq. (5) the prefactor 2 is due to spin degeneracy, whereas \( N(V) \), \( R(V) \), and \( R_A(V) \) are the number of transverse propagating modes (open channels) available at energy \( eV \) measured from the Fermi level \( \varepsilon_F \), the normal reflection coefficient, and the AR coefficient, respectively. Below the gap \( (eV < \Delta_0) \) only AR processes can contribute to the conductance since the unitarity of the scattering matrix implies that \( N - R = R_A \) in this range.

### III. RESULTS FOR THE IDEAL CASE

We present our numerical results, obtained using the numerical recursive Green’s function technique, in the physically realistic regime \( \delta \gg \Delta_0 \). In this section we focus on the ideal case of a GNR with a fixed number of sites along the transverse direction. We first discuss the case where the
also vanishes in this voltage range for even $N_W$. In contrast, for odd $N_W$, $G$ takes its maximum value in the same voltage range. In the regime $\Delta_0 < eV < \delta$, where quasiparticle transmission becomes possible, $R_A$ drops abruptly to zero also for odd $N_W$. This is due to the fact that the transmission processes, which are characterized by a small momentum transfer (intra- or inter-valley), largely dominate over AR processes, which instead involve large momentum transfer (intervalley). We thus find a finite conductance with a value equal to the number $N$ of open channels (in units of $2e^2/h$). Finally, for $eV > \delta$, $R_A$ is finite but quite small in both cases (for clarity $R_A$ has been multiplied by an enhancement factor) since voltages in this range are well above the superconducting gap.

These features ($R_A=0$ for $eV<\delta$ if $N_W$ is even and $R_A=1$ for $eV<\Delta_0$ if $N_W$ is odd) can be understood in terms of a pseudoparity superselection rule for the scattering states of the GNR. Let us consider a right-moving incoming electron (labeled by “1” in Fig. 2) with an energy lying between $e_F$ and the bottom of the second band. The only hole state available for an AR process is the one labeled by “2”, which is characterized by a pseudoparity $\eta$ that is (i) opposite to the one of the incoming electron for even $N_W$ and (ii) equal to the one of the incoming electron for odd $N_W$.

A superconducting parameter that is constant along the transverse direction cannot couple states with opposite pseudoparity (i.e., the pseudoparity of an electron impinging onto the SC interface cannot be flipped). Thus for even $N_W$ the AR process is forbidden and $R_A$ vanishes, whereas for odd $N_W$ this process is allowed and $R_A$ is finite. Notice that in the latter case normal reflection is forbidden by pseudoparity conservation, yielding $R_A=1$ in the subgap regime. For $\Delta_0 < eV < \delta$ normal transmission is the dominating process, so that $R_A$ is strongly suppressed for both even and odd $N_W$. A finite AR is restored for $eV > \delta$. Indeed in this case intravalley scattering into hole states with the appropriate pseudoparity [“4” in Figs. 3(a) and 3(b)] is available. This superselection rule is the ribbon counterpart of the bulk Klein-paradox selection rules based on pseudospin and also explains the valley-valve effect in $p$-$n$ junctions in GNRs.

**B. Finite gate voltage**

Let us now consider the case $U \neq 0$ (and $U \ll \delta$). In Fig. 6 we plot our results for $G$, $R_A$, and $N$.

A comparison between Figs. 5(a) and 6(a) shows that the application of a gate voltage has dramatic consequences on electron conduction in GNRs with even $N_W$. This is evident from Fig. 4(a). AR processes are allowed by pseudoparity conservation in the bias range $eV < U$, while they are forbidden for $U < eV < \delta$. This fact is reflected in the results shown in Fig. 6(a), where the differential conductance is finite (and equal to $4e^2/h$) for $eV < U$ and vanishes in the range $eV < \sqrt{U^2 + \Delta_0^2}$.

For subgap voltages electron conduction can be switched on and off by the application of a gate voltage. The junction between an even-$N_W$ GNR and a SC exhibits the operational behavior of an electron transistor based on the pseudoparity conservation law.

In contrast, for odd $N_W$ the application of a gate voltage is expected to have no major qualitative impact on the transport...
properties of a junction between a GNR with odd $N_W$ and a SC. Once again, this expectation can be traced back to the pseudoparity quantum number, as depicted in Fig. 4(b). The data reported in Fig. 6(b) show indeed the same qualitative behavior reported in Fig. 5(b). The only slight modification is that the drop of $R_A$ to zero, in the range $eV\approx \Delta_0$, is less abrupt than in Fig. 5(b) due to a slight momentum mismatch in the transmission channel between GNR and SC sides, arising when $U \neq 0$.

Before concluding we would like to comment on the double-spike structure of $R_A$ at voltages $eV=\delta \pm U$, which is seen in both panels of Fig. 6. This is due to the opening of additional channels for AR processes made available by the existence of the first-excited energy band (see Fig. 4).

IV. RESULTS FOR THE NONIDEAL CASE

In experimental realizations of GNRs the number of transverse sites varies along the longitudinal direction. Here we present a study of these “nonideal” effects on the predictions made in Sec. III. For the sake of generality, we focus our attention on the case $U \neq 0$.

We introduce a more realistic model for a GNR by altering the upper and the lower edge profiles with respect to the ideal case depicted in Fig. 1. A typical nonideal edge configuration we have considered is depicted in Fig. 7. The number of sites in the transverse direction is changed by one unit in a stepwise fashion, independently along each edge (see Fig. 7). The average step length is taken to be equal to $\bar{M}$ in both edges. The strip close to the interface with the SC has a length $M_1=\bar{M}$ and a width $N_{W,i}$. In what follows we will show that what really determines the behavior of the junction is the parity of this strip, provided that $M_i$ is sufficiently large.

In Fig. 8 we present numerical results for the nonlinear conductance for three values of $\bar{M}$ ($\bar{M}=4$, 6, and 8) and for a maximum width variation in four sites. Also in this case we present separately the results for even and odd $N_{W,i}$. In Fig. 8(a) ($N_{W,i}$ is even) we observe that the sharp drop of $R_A$ and $G$ at $eV=U$ that we had in the ideal case [see Fig. 6(a)] has changed into a smooth crossover over a bias interval $\delta V$. We find that $\delta V$ decreases with increasing $\bar{M}$; for $\bar{M}=8$ the conductance behavior is essentially identical to the ideal one. A similar behavior is observed for odd $N_{W,i}$; see Fig. 8(b). Note that for small $\bar{M}$ sharp antiresonances occur at energies $eV=U$.

Because $\delta V$ decreases with increasing $\bar{M}$, below we determine the minimum value of $\bar{M}$, $M^*$, which leads to a substantially ideal behavior. To do so, we analyze a simplified case in which the GNR is made up of two regions. The strip of length $M_i$ is connected to a region with a number of transverse sites that differs by one unit from $N_{W,i}$. The junction is said to be ideal if its AR coefficient satisfies the following inequality: $|R_A(V)−R_{A_{\text{ideal}}}(V)|<10^{-2}$ for every $V$ such that $|eV−U|/\Delta_0>10^{-2}$.

In Fig. 9 we report $M^*$ as a function of $\Delta_0$ (even case). Two comments are in order at this point: (i) since $M^* \ll N_{W,i}$, a relatively narrow strip close to the SC is sufficient to ensure the ideal behavior of the junction described in Sec. III and (ii) $M^*$ decreases with increasing $N_{W,i}$, saturating when $N_{W,i}=150$. The asymptotic value to which $M^*$ saturates (in the case of Fig. 9 it is roughly eight) depends on the specific criterion that one chooses. A similar scenario applies to the case of odd $N_{W,i}$. This curve supports the numerical results illustrated in Fig. 8; for a GNR with $N_{W,i}=100$ the
threshold length is roughly ten, explaining why the results corresponding to $M^* = 4$ depart substantially from ideality.

V. CONCLUSIONS

In summary, we have studied Andreev reflection in graphene nanoribbon/superconductor hybrid junctions. We have reported analytical expressions for the eigenfunctions of the tight-binding Hamiltonian describing the graphene nanoribbon, which carry explicitly a definite pseudoparity. The superselection rule stemming from this quantum number has two main implications on transport through the hybrid junction. For narrow nanoribbons with an even number of sites in the transverse direction, we have found a complete suppression of Andreev reflection in a range of energies that is huge when the Fermi energy lies at the Dirac level. This implies zero conductance at subgap voltages, which can however be restored by applying a finite gate potential, opening up potential technological applications of these hybrid junctions as electron transistors as well as nanorefrigerators. In contrast, in the case of narrow nanoribbons with an odd number of sites, we have found perfect Andreev reflection at subgap voltages and an abrupt suppression of it at supergap voltages.

We have also examined the case of nonideal nanoribbons in which the number of transverse sites varies along the transport direction. We have found that the transport properties of the junction in this case are determined by the parity of the region close to the interface with the superconductor. The behavior of the junction is indeed essentially ideal, provided that its length is larger than a certain threshold. These findings are relevant to realistic hybrid junctions between a graphene flake and a superconductor. Even for micrometer-sized flakes finite-size effects play a very important role since the confinement-induced mean energy spacing is larger than the superconducting gap.

The role of electron-electron interactions and/or next-nearest-neighbor hopping, which has not been addressed in the present work, can be qualitatively understood along the following lines. These effects have been shown to lead to the opening of a gap $\Delta_\text{e}$ at the Dirac level, which is however typically much smaller than $\delta$. Our conclusions thus remain valid for a large range of energies even when these effects are taken into account.

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FIG. 8. (Color online) The differential conductance $G$ in units of $2e^2/h$ is shown as a function of the applied bias $eV$ (in units of $\Delta_0$) for various values of $M$. (a) $N_{W,i}=100$. The (black) short-dashed line corresponds to $M=4$, whereas the (red) long-dashed line and the (green) solid line correspond to $M=6$ and $M=8$, respectively. (b) Same as in panel (a) but for $N_{W,i}=101$. For $M=8$ the ideal-GNR behavior depicted in Fig. 6 is almost recovered.

FIG. 9. (Color online) The threshold length $M^*$ as a function of $N_{W,i}$ (even case only).
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If one chooses $k_y \in [-\pi/(a\sqrt{3}), \pi/(a\sqrt{3})]$ then $s_A(y)=(-1)^{y_{M1}}$ for even $N_{M1}$ whereas $s_A=(-1)^{y_{M2}}$ and $s_B=(-1)^{y_{M1}+y_{M2}+1}$ for odd $N_{M2}$.

At higher energies and for the other bands the evanescent waves (3) become less localized at the edges (Ref. 28) and eventually turn into an oscillatory behavior when $k_y$ becomes imaginary. Nevertheless, the characterization of the bands via the pseudoparity quantum number holds at all energies, as depicted in Fig. 3.

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Notice that since graphene honeycomb lattice is bipartite, a simplified model in which $\Delta_i$ is assumed to be a step-like function along the $\hat{x}$ direction would effectively imply a rapid variation in $\Delta_i$ also transversally along the two-site cells. Since such situation would not be realistic, we have adopted a smooth variation in $\Delta_i$ over a length $\lambda=10a$ along the $\hat{x}$ direction.


34 The fact that $G$ vanishes in the range $\Delta_0 < eV < \sqrt{U^2+\Delta_0^2}$ can also be understood in terms of pseudoparity conservation.