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The aim of this paper is to study the time delay on electromagnetic signals propagating across a binary stellar system. We focus on the antisymmetric gravitomagnetic contribution due to the angular momentum of one of the stars of the pair. Considering a pulsar as the source of the signals, the effect would be manifest both in the arrival times of the pulses and in the frequency shift of their Fourier spectra. We derive the appropriate formulas and we discuss the influence of different configurations on the observability of gravitomagnetic effects. We argue that the recently discovered PSR J0737-3039 binary system does not permit the detection of the effects because of the large size of the eclipsed region.

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I. INTRODUCTION

Gravitomagnetic effects are perhaps the most elusive phenomena predicted by general relativity (GR). These effects are originated by the rotation of the source of the gravitational field, which gives rise to the presence of off-diagonal g_{0i} terms in the metric tensor. The gravitational coupling with the angular momentum of the source is indeed much weaker than the coupling with mass alone (gravito-electric interaction). Considering an axisymmetric stationary configuration, we may compare the relevant terms of the metric tensor by looking at the ratio

$$\varepsilon = \frac{g_{0\phi}}{g_{00}}, \qquad (1)$$

where a polar noncoordinated basis is assumed with unit forms $\omega^0 = c dt$ and $\omega^\phi = r d\phi$. Almost everywhere in the Universe a weak field approximation is acceptable, hence

$$g_{00} = 1 - \frac{R_S}{r}, \qquad (2)$$

$$g_{0\phi} = \frac{aR_S}{r^2} \sin^2\theta, \qquad (3)$$

where $R_S = 2GM/c^2$ is the Schwarzschild radius of the source, M being its mass; we have defined $a = J/(Mc)$ where J is the source angular momentum. In the equatorial plane ($\theta = \pi/2$), Eq. (1) reads

$$\varepsilon = \frac{aR_S}{r(r - R_S)} \simeq \frac{aR_S}{r^2}. \qquad (4)$$

Evaluation of Eq. (4) at the surface of the Sun (the most favorable place in the solar system) gives $\varepsilon \sim 10^{-12}$, thus evidencing the weakness of the gravitomagnetic versus the gravito-electric interaction.

The smallness of ε is the reason why, though having been suggested from the very beginning of the relativistic age [1], the explicit verification of the existence of grav-

itomagnetic effects has been extremely limited so far. Although a number of proposals for experimental tests of gravitomagnetism have been put forward during the past decades [2,3], the only one presently under way is the Gravity Probe B (GP-B) mission [4], which is currently collecting scientific data to verify both the Lense Thirring and the Schiff [5] precessions of orbiting gyroscopes. Other existing experimental tests of gravitomagnetism are:
 (i) lunar laser ranging [6];
 (ii) laser ranging of terrestrial artificial satellites LAGEOS and LAGEOS II [7].
Ratio (4) can be less unfavorable whenever r is approaching the Schwarzschild radius of the source: this can be the case of a source of electromagnetic (e.m.) signals orbiting around a compact, collapsed object, as it may happen in a compact binary system where (at least) one of the stars is a pulsar.

The aim of this paper is to discuss the time delay in the propagation of e.m. signals in GR. It is well known that the curvature of spacetime produces a delay in the propagation time of light with respect to a flat environment (Shapiro delay), a phenomenon that has been measured within the solar system [8–10].

In the presence of a rotating source, a specific gravitomagnetic contribution to the delay is also expected, which would show up as an asymmetry in the time of flight of the signals. In Ref. [11] it was proposed to look for this effect in the vicinity of the Sun. However, the magnitude of the effect is really tiny, $\sim 10^{-10}$ s from opposite sides of the solar disk. Other proposals pertain to the measurement of the frequency shift induced on e.m. signals by the gravitomagnetic field of the Sun [12].

As we pointed out above, a more favorable situation could be expected in a compact binary system. Presently, just a few of them are known [13], but all are interesting laboratories for testing GR. Among the known systems, the recently discovered PSR J0737-3039 [14] presents a favorable configuration and is also particularly appealing be-

cause both stars are pulsars. The data collection is going on and maybe some interesting results can also be found with respect to gravitomagnetic effects: for example, it has been recently argued that both the precession of the spinning bodies and the spin effects on the orbit could be measured in this system [15].

In this paper we derive the gravitational time delay on e.m. pulses in a binary system, focusing on the gravito-magnetic contribution, and propose how its consequences could be revealed. The corresponding frequency shift is also briefly discussed.

The organization of this paper is as follows. In Sec. II we describe the geometrical background and the hypotheses assumed to calculate the time delay. In Sec. III we review the properties of the binary system PSR J0737-3039, point-ing out its relevance for experimental tests of GR. In Sec. IV we apply the developed formalism to a PSR J0737-3039-like binary system and in Sec. V we present our conclusions.

II. TIME DELAY FROM A BINARY SYSTEM

We shall refer to two objects, composing the binary system, with notation $\mathcal{O}_1$ and $\mathcal{O}_2$. Object $\mathcal{O}_2$ is supposed to be rotating (e.g., a rotating neutron star) and is then the source of the gravitomagnetic field. The other object $\mathcal{O}_1$ (e.g., the radio pulsar) plays the role of the source of e.m. beams. Thus, we shall focus on time delay observed on the e.m. signals emitted by $\mathcal{O}_1$ when they experience the field generated by $\mathcal{O}_2$. Furthermore, we shall consider observers that are far away from the source of the gravitational field, so that they do not feel its effects, e.g., Earth-based ob-servers. In this case, the coordinate time corresponds to the proper time measured by the observers.

The derivation of the expressions relative to such a configuration relies on some standard assumptions, listed from (i) to (vii) in the following.

(i) We choose Cartesian coordinates, whose origin is located on the mass which is the source of the gravitomag-netic field (i.e., object $\mathcal{O}_2$): the z axis is aligned with the direction of the angular momentum $\vec{J}$ of the source (see Fig. 1), the x axis is orthogonal to the line of sight from Earth, and the y axis is orthogonal to both; we shall refer to xyz as to the "gravitomagnetic" reference frame. Consequently, the line element reads

$$ds^2 = g_{00}c^2dt^2 + g_{xx}dx^2 + g_{yy}dy^2 + g_{zz}dz^2$$
$$+ 2g_{0x}cdxdt + 2g_{0y}cdydt. \quad (5)$$

(ii) The gravitational field is in any case weak enough to admit the approximation ($r = \sqrt{x^2 + y^2 + z^2}$)

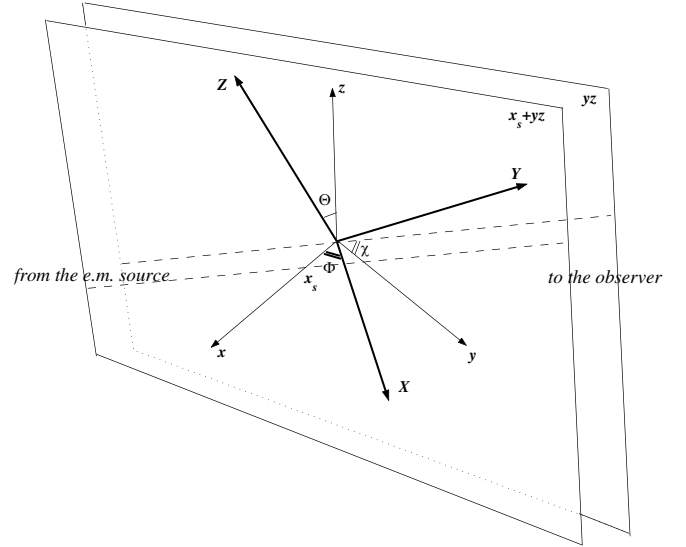$$g_{00} = 1 - \frac{R_S}{r}, \quad (6)$$

FIG. 1. Gravitomagnetic reference frame xyz. The origin is located on $\mathcal{O}_2$ and the z axis is aligned with the direction of its angular momentum. The Cartesian reference frame XYZ is located in $\mathcal{O}_2$ and XY identifies the orbital plane of the binary system.

$$g_{xx} = g_{yy} = g_{zz} = -1 - \frac{R_S}{r}, \quad (7)$$

$$g_{0x} = -\frac{aR_S y}{r^3}, \quad (8)$$

$$g_{0y} = \frac{aR_S x}{r^3}. \quad (9)$$

(iii) The trajectory of light rays is assumed to be a straight line. Actually, there is a bending, whose effects are usually assumed to be negligible [16,17].

(iv) The center of mass of the system is at rest with respect to the observer on Earth.

(v) The orbit of the source of the signals (i.e., object $\mathcal{O}_1$) around the center of mass of the system is circular [18].

(vi) The size of the binary system is much smaller than the distance from the observer on Earth.

(vii) As a result of the proper motion of object $\mathcal{O}_2$, the origin of the reference frame is moving with respect to the center of mass of the system and then with respect to the observer. However, we shall assume that this motion is slow enough not to appreciably change the expression (8) of the metric. In practice, from the viewpoint of the ob-server, the propagation of light through the system is described as a series of static snapshots.

Under these conditions, we identify the position of the source of the signals with the space coordinates (x_s, y_s, z_s) and that of the observer with (x_s, y_{obs}, z_{obs}). According to hypothesis (iii), the trajectory of the e.m. beam is a straight line,

$$x = x_s, \qquad (10)$$

$$z = z_s + (y - y_s)\tan\chi. \qquad (11)$$

Each beam, represented by a dashed line in Fig. 1, lies in a plane parallel to yz, thus

$$0 = g_{00}c^2 dt^2 + g_{yy}h^2 dy^2 + 2g_{0y}cdydt, \qquad (12)$$

and in the components of the metric tensor (6)–(9) we can replace $r^2 = h^2 y^2 + 2ky + w^2$ with

$$h = \sqrt{1 + \tan^2\chi}, \qquad (13)$$

$$k = (z_s - y_s \tan\chi)\tan\chi, \qquad (14)$$

$$w = \sqrt{(z_s - y_s \tan\chi)^2 + x_s^2}. \qquad (15)$$

According to the standard approach to the time-delay problem [16], we solve Eq. (12) for dt/dy; then the result is integrated along the trajectory of the ray; i.e.,

$$t_{\text{flight}} = \frac{1}{c}\int_{y_s}^{y_{\text{obs}}} dy(r - R_S)^{-1}\left\{-\frac{aR_S x_s}{r^2}\right.$$
$$\left. + \sqrt{\frac{a^2 R_S^2 x_s^2}{r^4} + (r^2 - R_S^2)h^2}\right\}. \qquad (16)$$

When the propagation is "on the left" of the oriented projection of $\vec{J}$ in the sky, with respect to the observer ($x_s > 0$; see Fig. 1), the first term in the parentheses is negative; on the opposite, when the propagation is on the right ($x_s < 0$), the sign is positive.

For the weak field condition of (ii), Eq. (16) can be further expanded in powers of R_S and a, up to their product, i.e., up to second order. Second order is necessary to describe the gravitomagnetic interaction; in addition, it should be noticed that, for collapsed objects (as for a star like the Sun) it is reasonably $R_S \sim a$, so that the second order term in R_S cannot be simply neglected. By performing such an expansion, the integral of Eq. (16) is divided into four terms, which may be further grouped into three contributions to the total time of flight as

$$\delta t \equiv t_{\text{flight}} = t_0 + t_M + t_J, \qquad (17)$$

where

$$t_0 = \frac{1}{c}\int_{y_s}^{y_{\text{obs}}} hdy = \frac{h}{c}(y_{\text{obs}} - y_s) \qquad (18)$$

represents the pure geometric term, and

$$t_M = \frac{h}{c}R_S \int_{y_s}^{y_{\text{obs}}}\left(\frac{1}{r} + \frac{R_S}{2r^2}\right)dy$$
$$= \frac{R_S}{c}\ln\frac{h^2 y_{\text{obs}} + k + hr_{\text{obs}}}{h^2 y_s + k + hr_s} + \frac{1}{2c}\frac{hR_S^2}{\sqrt{h^2 w^2 - k^2}}$$
$$\times \arctan\left(2\frac{k + h^2 y^2}{\sqrt{h^2 w^2 - k^2}}\right)\Big|_{y_s}^{y_{\text{obs}}} \qquad (19)$$

is the mass delay up to second order, where we have defined $r_{\text{obs}} \equiv r(y_{\text{obs}})$ and $r_s \equiv r(y_s)$; the contribution due to the angular momentum of the source reads

$$t_J = -\frac{x_s}{c}\int_{y_s}^{y_{\text{obs}}}\frac{aR_S}{r^3}dy = \frac{x_s}{c}\frac{aR_S}{k^2 - h^2 w^2}\frac{h^2 y + k}{r}\Big|_{y_s}^{y_{\text{obs}}}. \qquad (20)$$

The quantity x_s changes sign if the e.m. source is on the left or on the right of the rotating body with respect to the observer; as a result, t_J can have opposite signs on opposite sides accordingly.

Focusing on the geometry peculiar of a binary system, one is expecting y_{obs} to be the sum of a time-independent part, y_0, corresponding to the distance from the center of mass of the system to the observer, and a time-dependent part, a contribution oscillating in time due to the orbital motion of object $\mathcal{O}_2$; condition (v) implies that this orbit, too, is a circumference of radius R_2. The amplitude of the oscillation is of the order of magnitude of the size of the binary system, just as k and w are. Since for condition (vi) the size of the system is much smaller than the distance from Earth, Eqs. (19) and (20) are simplified as

$$t_M \simeq \frac{R_S}{c}\ln\frac{2y_0 h^2}{h^2 y_s + k + hr_s} + \frac{1}{2c}\frac{hR_S^2}{\sqrt{h^2 w^2 - k^2}}$$
$$\times\left\{\arctan\frac{2h^2 y_0^2}{\sqrt{h^2 w^2 - k^2}} - \arctan\frac{2(k + h^2 y_s^2)}{\sqrt{h^2 w^2 - k^2}}\right\}, \qquad (21)$$

$$t_J \simeq \frac{x_s}{c}\frac{aR_S}{k^2 - h^2 w^2}\left(h - \frac{h^2 y_s + k}{r_s}\right), \qquad (22)$$

where we have set $h^2 y_s^2 + 2ky_s + w^2 = r_s^2 = R^2$; R is the distance between the two stars in the pair, which is constant according to hypothesis (v). By restoring an explicit notation, we have

$$t_M = t_{M_1} + t_{M_2}$$

$$\simeq \frac{R_S}{c} \ln \frac{2y_0}{(y_s \cos\chi + z_s \sin\chi + R)\cos\chi}$$

$$+ \frac{1}{2c} \frac{R_S^2}{\sqrt{R^2 - (y_s \cos\chi + z_s \sin\chi)^2}}$$

$$\times \left(\frac{\pi}{2} - \arctan \frac{2(y_s \cos\chi + z_s \sin\chi)}{\sqrt{R^2 - (y_s \cos\chi + z_s \sin\chi)^2}} \right), \quad (23)$$

$$t_J \simeq -\frac{x_1}{c} \frac{aR_S}{R} \frac{\cos\chi}{R + y_s \cos\chi + z_s \sin\chi}. \quad (24)$$

A. The time-dependent part

In Eqs. (18), (23), and (24) we are interested in the time-dependent part, which is implicit in x_s, y_s, and z_s, since the position of the e.m. source at each successive "snapshot" is different because of the orbital motion of $\mathcal{O}_1$. By assumption (v), the orbit of $\mathcal{O}_1$ is circular. Let us start by expressing the position of $\mathcal{O}_1$ with respect to another reference frame (called XYZ; see Fig. 1) centered in $\mathcal{O}_2$ such that $X = R\cos\omega t$ and $Y = R\sin\omega t$, where ω is the orbital angular velocity of the pair and the X axis is identified by the intersection between the orbital plane of the system and the gravitomagnetic equatorial plane of $\mathcal{O}_2$. We call Φ the angle between the X axis and the x axis; Θ identifies the tilt angle between the axis of the orbit and the angular momentum $\vec{J}$ of $\mathcal{O}_2$. The gravitomagnetic coordinates of $\mathcal{O}_1$ expressed with respect to the xyz frame read

$$\xi_s = \cos\Phi\cos\psi - \cos\Theta\sin\Phi\sin\psi,$$

$$\eta_s = \sin\Phi\cos\psi + \cos\Theta\cos\Phi\sin\psi, \quad (25)$$

$$\zeta_s = -\sin\Theta\sin\psi,$$

where, for convenience, we have introduced the reduced coordinates $\xi_s = x_s/R$, $\eta_s = y_s/R$, $\zeta_s = z_s/R$, and we have defined $\psi = \varphi + \alpha$: $\varphi = \omega t$ is the orbital phase and $\alpha = \arctan(\cot\Phi/\cos\Theta)$. We can measure times from the configuration $x_s = 0$, $y_s > 0$ (conjunction).

From Eq. (18), the time-dependent contribution t_0^* in flat spacetime reads (hereafter, starred quantities refer to the time-dependent contributions to t_{flight})

$$\frac{ct_0^*}{r - R}\cos\chi = \sin\Phi\cos\psi + \cos\Theta\cos\Phi\sin\psi, \quad (26)$$

that is, a harmonic oscillation whose amplitude corresponds to the time the beam takes to cross the system.

The mass-dependent term is more involved, since it is composed of a first order and a second order term. Both of them can be factorized into an "amplitude," containing the size of the system and the Schwarzschild radius of $\mathcal{O}_2$, and a pure geometrical part. It is given by the sum $t_M^* = t_{M_1}^* + t_{M_2}^*$, with

$$\frac{ct_{M_1}^*}{R_S} = -\ln(\eta_s \cos\chi + \zeta_s \sin\chi + 1), \quad (27)$$

$$\frac{ct_{M_2}^*}{R_S^2} = \frac{1}{2R\sqrt{1 - (\eta_s \cos\chi + \zeta_s \sin\chi)^2}}$$

$$\times \left(\frac{\pi}{2} - \arctan \frac{2(\eta_s \cos\chi + \zeta_s \sin\chi)}{\sqrt{1 - (\eta_s \cos\chi + \zeta_s \sin\chi)^2}} \right). \quad (28)$$

Eventually, from Eq. (24), the contribution to the time delay due to the angular momentum of $\mathcal{O}_2$ reads

$$\frac{ct_J}{aR_S}\frac{R}{\cos\chi} \simeq -\frac{\xi_s}{(1 + \eta_s \cos\chi + \zeta_s \sin\chi)}. \quad (29)$$

These time-dependent parts of t_{flight} would be visible in the sequence of the arrival times of the pulses from the source $\mathcal{O}_1$.

III. THE BINARY SYSTEM PSR J0737-3039

The recently discovered binary system PSR J0737-3039 has proved to be an important laboratory for testing relativistic theories and, in principle, it could also be useful for measuring gravitomagnetic effects on time delay. Let us then briefly review its most important physical features [14]. The two pulsars, J0737-3039A and J0737-3039B (hereafter simply A and B), have periods $P_A = 23$ ms and $P_B = 2.8$ s; they revolve about each other in a 2.4-hr orbit of significant eccentricity (0.088); the separation of the two objects is typically 9×10^5 km. The orbital plane is viewed nearly edge-on from the Earth, with an inclination angle of $i = 87° \pm 3°$. It has been possible to detect a huge rate of periastron advance, $\dot{\omega} = 16.88°$ yr^{-1}, which is about 4 times the one of PSR 1913 + 16 [19]. If this effect is entirely due to GR, from the observations carried out so far it has been possible to establish that $M_A = 1.337(5)M_\odot$ and $M_B = 1.250(5)M_\odot$. In addition, due to the collision of A's wind with B's magnetosphere it seems very likely that the spin axis of B is aligned with the orbital angular momentum of the system [20,21]. On the other hand, observations show that A is almost an aligned rotator (angle between A's magnetic and rotation axes $\sim 5°$ [20]), but with its spin axis substantially misaligned with the orbital angular momentum by $\sim 50°$. In addition, the system has the important feature that, for 27 s, A is eclipsed by B's magnetosphere. Such duration of the eclipse was used in Ref. [22] to place a limit of 18.6×10^3 km on the size of the eclipsed region. This region is much bigger than the expected physical linear dimensions of an actual neutron star (~ 10 km): the typical features of the observed signals seem to suggest that the eclipse is due to the absorption of the radio emission from A by a magneto-sheath surrounding B's magnetosphere [21,23,24].

Because of (i) the alignment of the orbital plane with the line of sight, (ii) the fact that B eclipses A, and (iii) the spin axis of B is probably perpendicular to the orbital plane, the

TABLE I. A toy model for system PSR J0737-3039. We choose $\omega \sim 7 \times 10^{-4}$ s^{-1} and $R = 2R_2 \simeq 10^9$ m. From left to right, the columns contain which star of the pair is the source of the gravitomagnetic field, the Schwarzschild radius R_S of $\mathcal{O}_2$, and the angles that identify the geometrical configuration of the system.

$\mathcal{O}_1$	$\mathcal{O}_2$	R_S (m)	χ	Θ	Φ
A	B	1.6×10^3	$0°$	$0°$	$0°$
B	A	1.7×10^3	$50°$	$50°$	$0°$

configuration of the system could be favorable for studying the gravitomagnetic effects on signals propagation.

IV. APPLICATION TO A PSR J0737-3039-LIKE MODEL

A. The time delay

Let us specify the simple model outlined above using values of the parameters similar to those of the actual PSR J0737-3039: star B is acting as the source of gravitomagnetism ($\mathcal{O}_2 \equiv B$ and $\mathcal{O}_1 \equiv A$; see Table I), $R \sim 10^9$ m and $\omega \sim 7 \times 10^{-4}$s^{-1}. The angular momentum of star B is aligned with the orbital angular momentum of the system ($\Theta = 0$) and we choose the most favorable configuration with $\Phi = 0$ (i.e., $\alpha = \pi/2$). The e.m. beams are thus propagating in the orbital plane ($\chi = 0$). Supposing that the progenitor star was only a little bigger than the Sun, and that most of the angular momentum was preserved during the collapse, we can assume $a \sim 10^3$ m. With these hypotheses, Eqs. (26)–(29) read

$$t_0^* = \mathcal{A}_0^B \cos\varphi, \tag{30}$$

$$t_{M_1}^* = \mathcal{A}_{M_1}^B \ln(1 + \cos\varphi), \tag{31}$$

$$t_{M_2}^* = \mathcal{A}_{M_2}^B \frac{1}{|\sin\varphi|}\left(\frac{\pi}{2} - \arctan\frac{2\cos\varphi}{|\sin\varphi|}\right), \tag{32}$$

$$t_J = \mathcal{A}_J^B \frac{\sin\varphi}{1 + \cos\varphi} \tag{33}$$

where the numerical coefficients $\mathcal{A}^{\mathcal{O}_2}$ with $\mathcal{O}_2 \equiv B$ give the order of magnitude of the effect (see Table II). The φ-dependent parts in Eqs. (31) and (33) become bigger and bigger close to the conjunction position ($\varphi = \pi$; i.e., when the impact parameter is zero), but this divergence has no physical meaning because the actual compact objects have finite dimensions and the beam cannot pass through the center of B.

Let us suppose that it is possible to identify conjunction ($\varphi = \pi$) and opposition ($\varphi = 2\pi$) points in the sequence of arriving pulses. Since the geometric and mass terms are symmetric with respect to conjunction and opposition, whereas t_J is antisymmetric, for $0 \leq \varphi \leq \pi$ we have
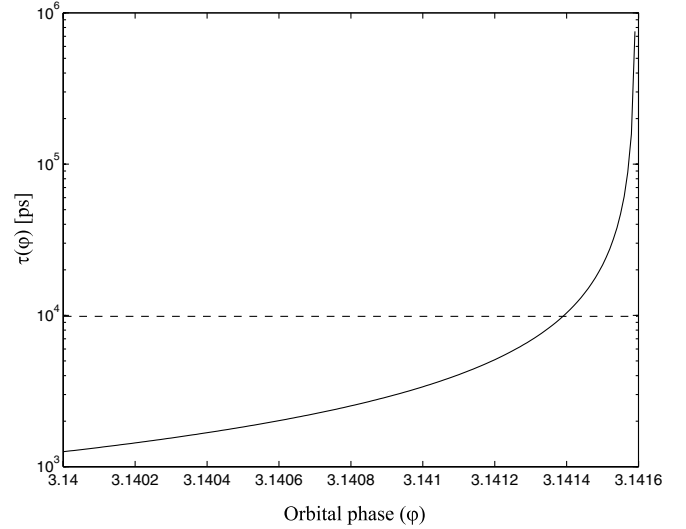

FIG. 2. The function $\tau(\varphi)$ (measured in picoseconds) versus the orbital phase of star A in proximity to the occultation position $\varphi = \pi$. The horizontal dashed line corresponds to the detectability threshold of 10^{-8} s.

$$\tau(\varphi) = \delta t^*(\varphi) - \delta t^*(2\pi - \varphi) = 2t_J, \tag{34}$$

i.e., in seconds

$$\tau(\varphi) \simeq 10^{-12} \frac{\sin\varphi}{1 + \cos\varphi}. \tag{35}$$

The function $\tau(\varphi)$ is shown in Fig. 2 (solid line) close to the conjunction position. If we suppose that the threshold for detecting the change in the arrival rate of the signals is, for instance, at 10^{-8} s, then only the part of the graph above the horizontal dashed line is useful. This means that the access to the interesting region would be possible only if the minimum impact parameter was smaller than $\sim 1.8 \times 10^2$ km. Since the typical radius of a neutron star is ~ 10 km, we can expect the appropriate conditions to be satisfied in a double pulsar binary system. However, this is not the case of PSR J0737-3039: in fact, in this system the minimum impact parameter is not given by the radius of object B, but rather by the size of the opaque area of the

TABLE II. The first two rows show the order of magnitude of the contributions to the time-dependent part of the time of flight of e.m. signals emitted by object $\mathcal{O}_1$ and propagating in the gravitational field generated by $\mathcal{O}_2$. The bottom row contains the order of magnitude of the contributions to the (relative) frequency shift on the signals emitted by star A.

$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{A}_0^{\mathcal{O}_2}$ (s)	$\mathcal{A}_{M_1}^{\mathcal{O}_2}$ (s)	$\mathcal{A}_{M_2}^{\mathcal{O}_2}$ (s)	$\mathcal{A}_J^{\mathcal{O}_2}$ (s)
A	B	-3	-5×10^{-6}	4×10^{-12}	6×10^{-13}
B	A	-4.7	-5.6×10^{-6}	4.8×10^{-12}	-3.6×10^{-12}

		$\mathcal{D}_0^{\mathcal{O}_2}$	$\mathcal{D}_{M_1}^{\mathcal{O}_2}$	$\mathcal{D}_{J_1}^{\mathcal{O}_2}$	$\mathcal{D}_{J_2}^{\mathcal{O}_2}$
A	B	-21×10^{-4}	-3.7×10^{-9}	-4×10^{-15}	4×10^{-15}

magnetosheath surrounding B itself, that is, 1.8×10^4 km; i.e., 2 orders of magnitude bigger than the detectability threshold.

The same kind of analysis can be performed by exchanging A with B; i.e., $\mathcal{O}_2 \equiv$ A and $\mathcal{O}_1 \equiv$ B, although in the actual system the B pulses are extremely weak and aleatory. In this case, we must choose $\Theta = 50°$ and $\chi = 50°$ (see Table I) since only the e.m. beams propagating in the orbital plane can be seen by the observer on Earth. In this case we have

$$t_0^* = \mathcal{A}_0^A \cos\varphi, \tag{36}$$

$$t_{M_1}^* = \mathcal{A}_{M_1}^A \ln(1 - n\cos\varphi), \tag{37}$$

$$t_{M_2}^* = \mathcal{A}_{M_2}^A \frac{1}{\sqrt{1 - (n\cos\varphi)^2}}\left(\frac{\pi}{2} - \arctan\frac{m\cos\varphi}{\sqrt{1 - (n\cos\varphi)^2}}\right), \tag{38}$$

$$t_J = \mathcal{A}_J^A \frac{\sin\varphi}{1 + n\cos\varphi}, \tag{39}$$

where $n = 0.17$ and $m = 0.34$, from the evaluation of Eqs. (26)–(29) employing the parameters of Table I. In this case, we obtain

$$\tau(\varphi) \simeq -7.2 \times 10^{-12} \frac{\sin\varphi}{1 + n\cos\varphi}. \tag{40}$$

Since $|n| < 1$, the denominator never vanishes, so that the amount of the effect is at most $\sim 10^{-11}$ s. This configuration is then less favorable than the previous one, because a lower detectability threshold is required.

B. The frequency shift

Let us consider the Fourier spectrum of an e.m. beam propagating into a gravitational field: the time-delay effect we have discussed so far corresponds, in the frequency domain, to a frequency shift of each harmonic component of the signal. Since the period T of each harmonic is much shorter than all the other characteristic time scales of the system, we can write its relative change in period, hence in the frequency ν, as

$$\frac{\delta\nu}{\nu} = -\frac{\delta T}{T} = -(\dot{t}_0 + \dot{t}_M + \dot{t}_J), \tag{41}$$

where the overdot stands for derivative with respect to the coordinate time. For each contribution, we have

$$\frac{\delta\nu}{\nu}\bigg|_0 = \omega\frac{h}{c}(R_2 - R)\sin\varphi, \tag{42}$$

$$\frac{\delta\nu}{\nu}\bigg|_{M_1} = \frac{R_S}{c}\frac{\dot{\eta}_s\cos\chi + \dot{\zeta}_s\sin\chi}{1 + \eta_s\cos\chi + \zeta_s\sin\chi}, \tag{43}$$

$$\frac{\delta\nu}{\nu}\bigg|_{M_2} = -(\dot{y}_s\partial_{y_s} + \dot{z}_s\partial_{z_s})t_{M_2}, \tag{44}$$

$$\frac{\delta\nu}{\nu}\bigg|_J = \frac{aR_S}{cR}\left\{\frac{\dot{\xi}_s\cos\chi}{1 + \eta_s\cos\chi + \zeta_s\sin\chi} - \frac{\xi_s(\dot{\eta}_s\cos^2\chi + \dot{\zeta}_s\sin^2\chi)}{(1 + \eta_s\cos\chi + \zeta_s\sin\chi)^2}\right\}, \tag{45}$$

where $\dot{\xi}_s$, $\dot{\eta}_s$, and $\dot{\zeta}_s$ are obtained from Eqs. (25). Here it becomes clear that the relative frequency shift is a complicated function of time that reads out as a periodic modification in the frequency spectrum of the signal.

Applying the above equations to our PSR J0737-3039-like system, we just present results for pulses emitted by star A. In this case, we have

$$\frac{\delta\nu}{\nu}\bigg|_0 = \mathcal{D}_0^B \sin\varphi, \tag{46}$$

$$\frac{\delta\nu}{\nu}\bigg|_M = \mathcal{D}_{M_1}^B \frac{\sin\varphi}{1 + \cos\varphi} - \dot{y}_s\partial_{y_s}t_{M_2}, \tag{47}$$

$$\frac{\delta\nu}{\nu}\bigg|_J = \mathcal{D}_{J_1}^B \frac{\cos\varphi}{1 + \cos\varphi} + \mathcal{D}_{J_2}^B \frac{\sin\varphi}{(1 + \cos\varphi)^2}, \tag{48}$$

where the coefficients $\mathcal{D}^{\mathcal{O}_2}$, with $\mathcal{O}_2 \equiv$ B, give the order of magnitude of the effect and are listed in Table II.

All the contributions (including $\delta\nu/\nu|_{M_2}$, that is then needless to make explicit) are odd in φ except for the one proportional to $\mathcal{D}_{J_1}^B$, which is even. Summing shifts symmetric with respect to the opposition point we obtain

$$\frac{\delta\nu}{\nu}(\varphi) + \frac{\delta\nu}{\nu}(2\pi - \varphi) = -8 \times 10^{-15}\frac{\cos\varphi}{1 + \cos\varphi}. \tag{49}$$

V. CONCLUSIONS

In this paper, we have discussed the effects of the gravitational interaction on the time delay of electromagnetic signals coming from a binary system composed of a radio pulsar and another compact object. In particular, we have evidenced that the behavior of the gravitomagnetic contribution, near the occultation of the radio pulsar by its companion, is antisymmetric while the geometric and mass contribution are symmetric, thus suggesting a possible way for decoupling the effects.

The recently discovered binary pulsar system PSR J0737-3039, lends itself to the study of the time delay because (i) the orbital plane almost contains the line of sight, (ii) the star B eclipses A, and (iii) the spin axis of B seems to be aligned with the orbital angular momentum.

However, we have argued that the gravitomagnetic effect on time delay still remains extremely small in this system: under reasonable assumptions on the mass and angular momentum of the sources of the gravitational field,

the possibility to reveal the effect critically depends on the configuration of the system and on the minimum impact parameter achievable for the e.m. ray. Because of the existence of a large opaque region represented by a magnetosheath surrounding PSR J0737-3039B, the effective impact parameter is much bigger than the actual linear dimension of a neutron star, so that the magnitude of the gravitomagnetic time delay is smaller than a reasonable detectability threshold.

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[17] In fact, (a) The increase in the time of flight purely due to the geometric lengthening of light rays because of the bending induced by curvature is relevant only within the binary system and is quadratic in the deviation angle α. The deviation angle is in turn proportional to the ratio between the mass of one of the stars and the impact parameter, which gives, in practice, a quadratic factor in the order of 10^{-9}; (b) The relevant quantity is the interval between two consecutive pulses from the source. The influence of the bending would depend on the change of the deviation angle during one period of the signal. In practice, this change introduces another factor $\sim 10^{-4}$ or less. The final impact is then less than one part in 10^{13}; (c) The effect due to the mass is symmetric with respect to conjunction; then it is irrelevant for gravitomagnetic contributions; (d) There is an anisotropy in the deviation angle too, but this effect is smaller than the very deviation.

[18] This condition is not really necessary, but simplifies the description of the system.

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