Tutorial lecture on Characterization and Macromodeling of 3D Interconnects

Characterization and Macromodeling of 3D Interconnects

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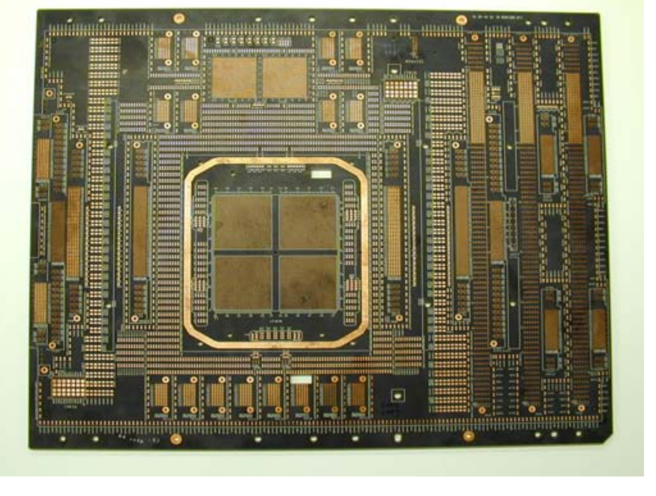
Introduction



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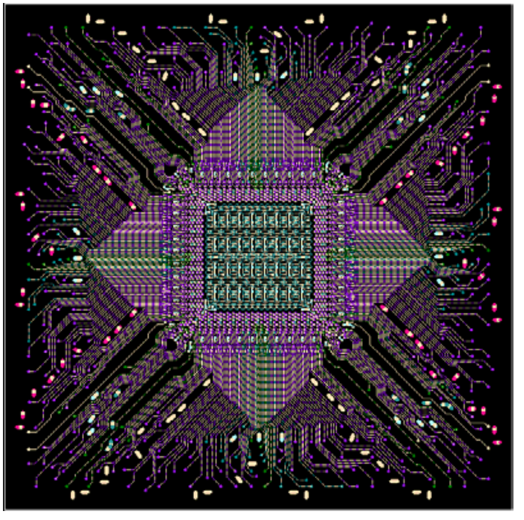
Introduction



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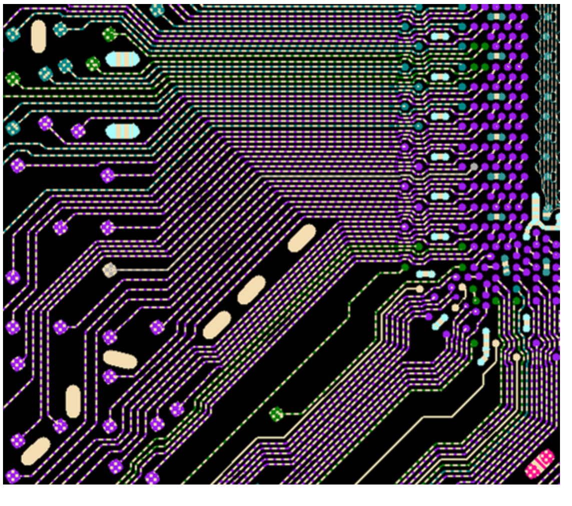

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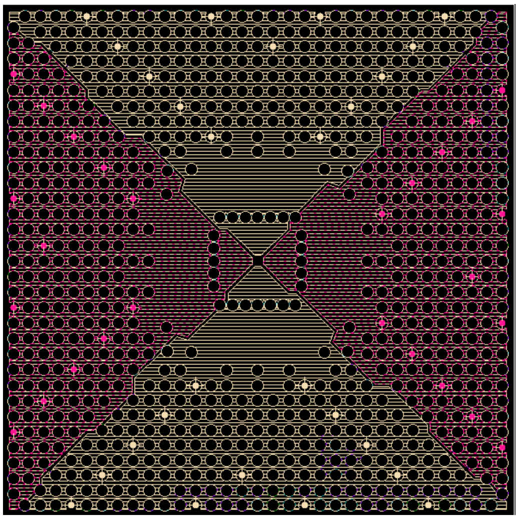

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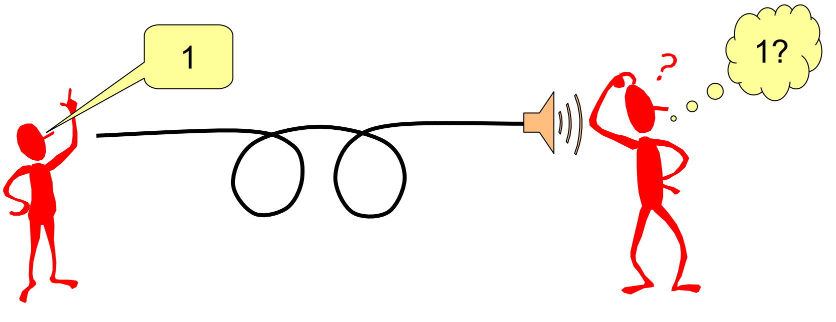
Introduction

High-speed Data transmission requires

integrity of the signals

thru lines, bends, vias, connectors, …



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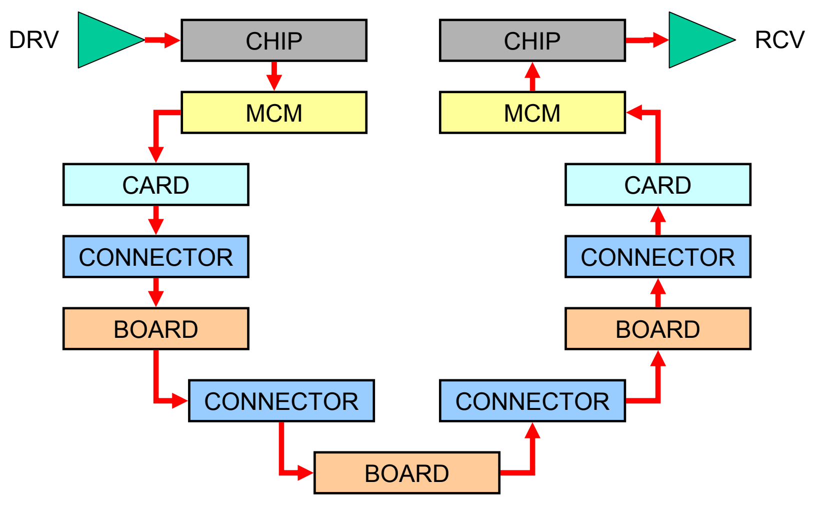
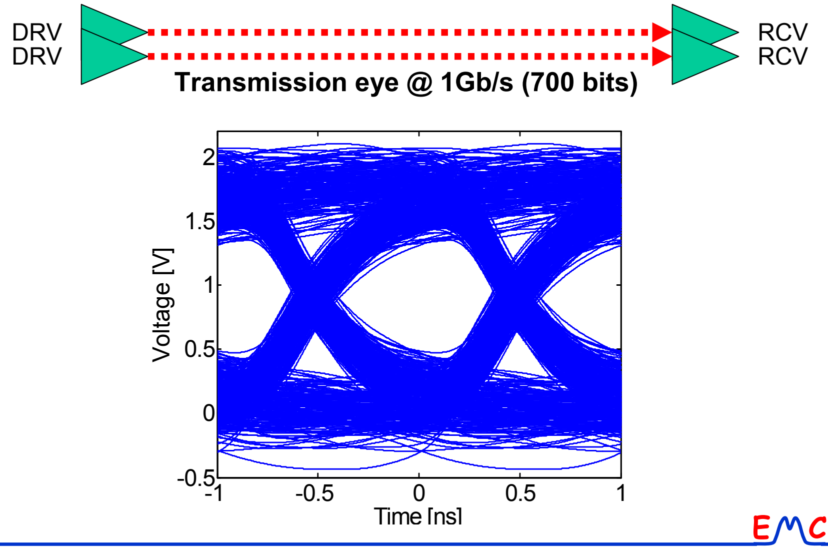
An example

An example



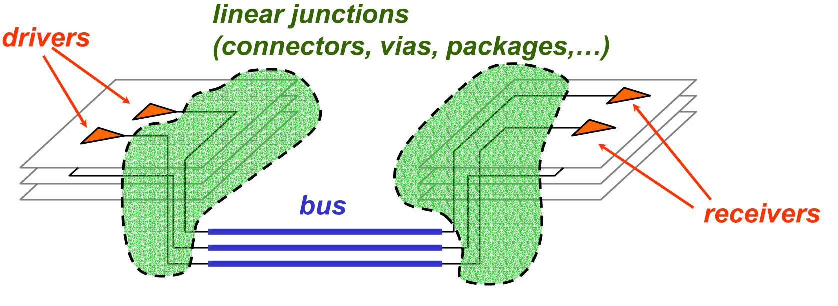
An example



Introduction

Signal Integrity issues in high-speed digital systems

Crosstalk, couplings, reflections, losses, dispersion, attenuation, resonances, ground noise, nonlinear effects, radiation, EMI, …



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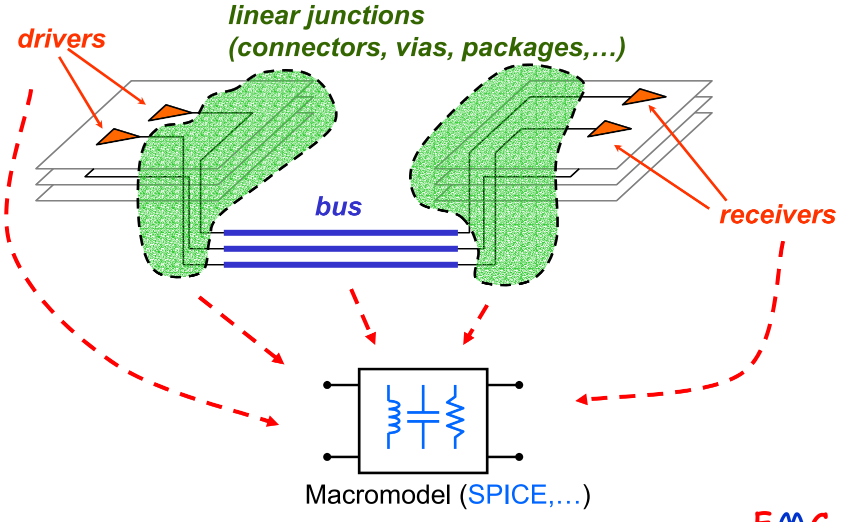
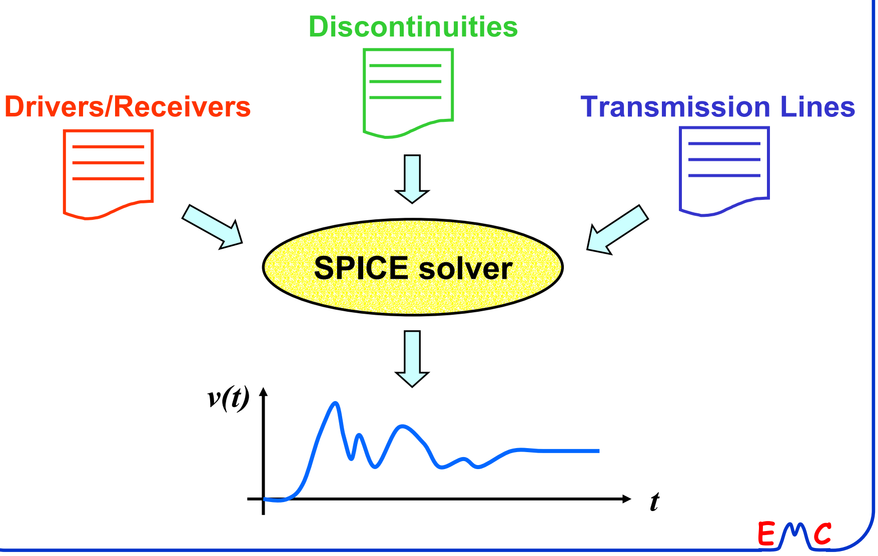
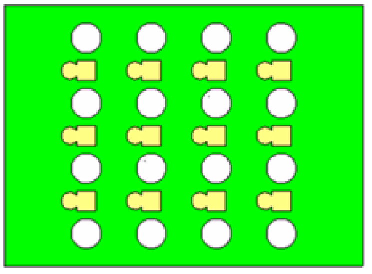
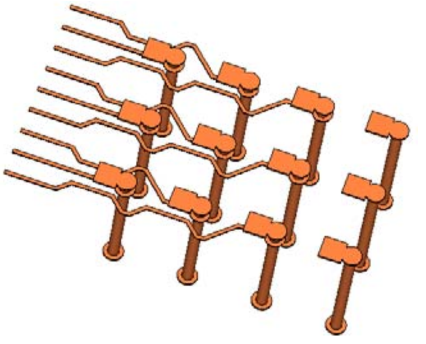


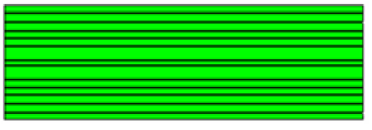
“Lego” approach

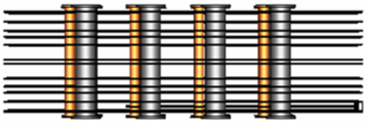


3D Interconnects









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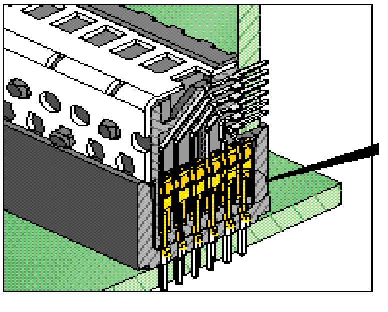
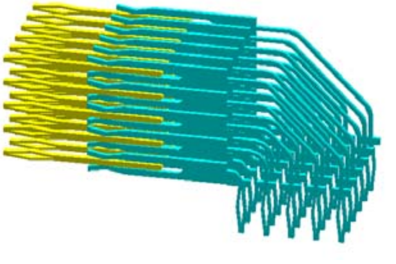
3D Interconnects





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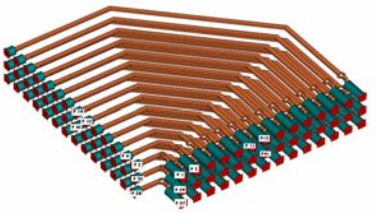
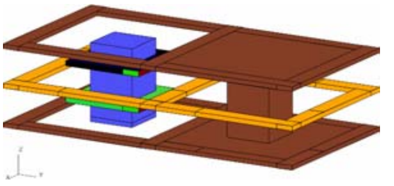
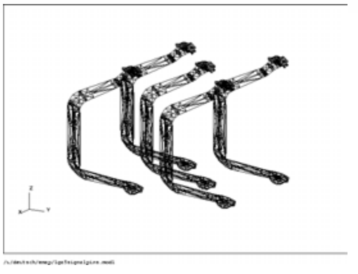
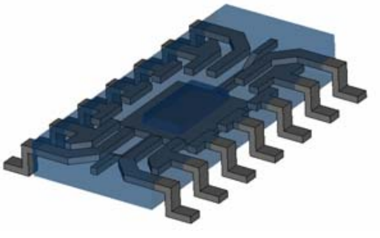
3D Interconnects









Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

Macromodeling approaches

Macromodeling of 3D interconnects for Signal Integrity

1. Capture physical effects leading to signal degradation
   - Must take into account 3D electromagnetic fields
     - Simulation or measurement
     - Many different characterizations are possible!
2. Use this information to build a macromodel
     - Many macromodeling approaches available!

Macromodeling approaches

Characterization via equations

Discretization of Maxwell full-wave equations

Model Order Reduction methods: build a simplified model from an existing (large) one

Characterization via port responses (Black Box)

Time or frequency domain

Simulated or measured

Reduced-Order Model Identification methods: build a model from samples of the port responses

Macromodeling approaches

Main goal of all (lumped) macromodeling methods:

produce a rational approximation

$$\mathbf{H}_q(s) = \mathbf{H}_\infty + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

Lumped circuits

- **have rational transfer functions**
- **are governed by Ordinary Differential Equations**

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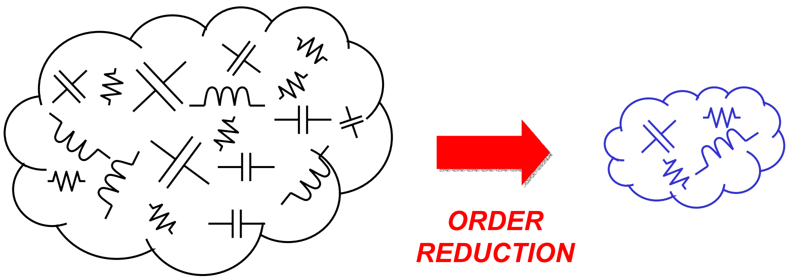
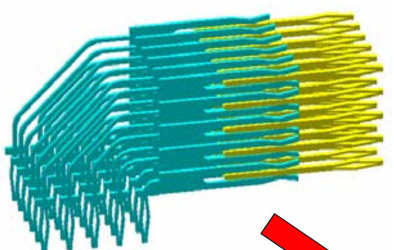
Moder Order Reduction

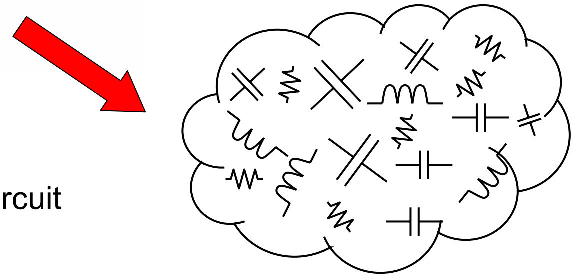


$$\mathbf{H}(s) \approx \mathbf{H}_q(s)$$

Possible scenarios

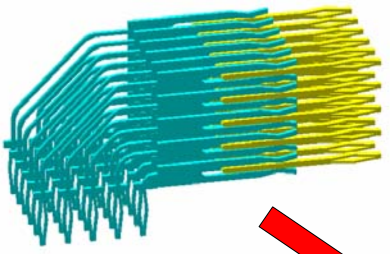


PEEC (Partial Element Equivalent Circuit) discretization



Large circuit

Possible scenarios

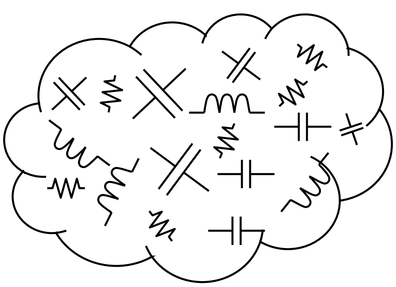


Spatial discretization of Maxwell equations (FDTD, FEM, MoM, …)

Large system

Set of ODEs (Ordinary Differential Equations)

Possible scenarios



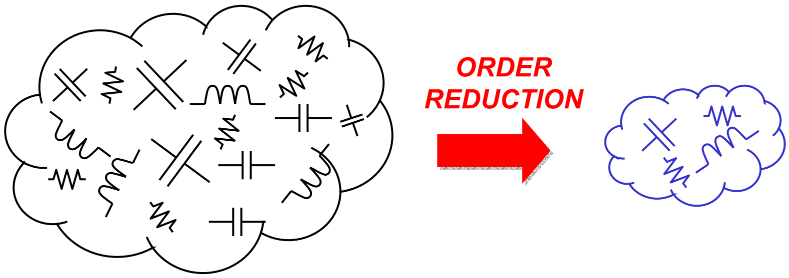
Set of ODEs (Ordinary Differential Equations)

MNA

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^{\mathrm{T}}\mathbf{x} \end{cases}$$

$$\mathbf{H}(s) = \mathbf{L}^{\mathrm{T}}(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B}$$

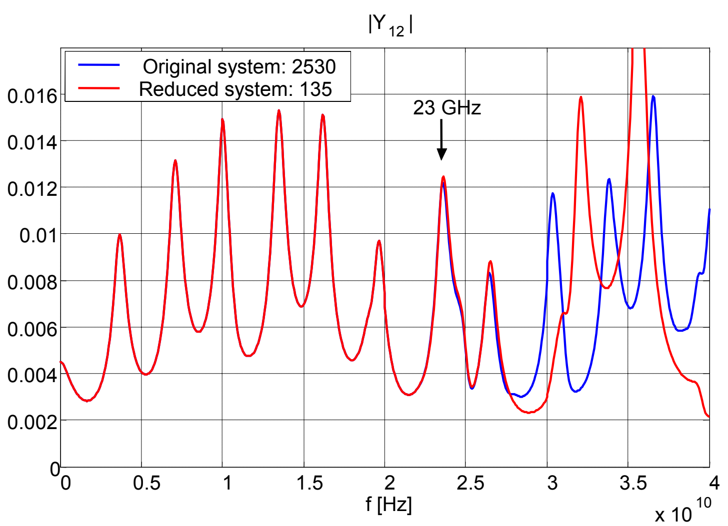
Approximation via moment matching



$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots + \mathbf{M}_N s^N + \cdots$$

$$\mathbf{H}_q(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots + \mathbf{M}_q s^q + \cdots$$

Moment matching: an example



Moments

$$\begin{cases} \mathbf{Gx} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{Bu} \\ \mathbf{y} = \mathbf{L}^{\mathrm{T}}\mathbf{x} \end{cases} \qquad \begin{cases} \mathbf{x} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{Ru} \\ \mathbf{y} = \mathbf{L}^{\mathrm{T}}\mathbf{x} \end{cases} \qquad \begin{matrix} \mathbf{A} = -\mathbf{G}^{-1}\mathbf{C} \\ \mathbf{R} = \mathbf{G}^{-1}\mathbf{B} \end{matrix}$$

$$\mathbf{H}(s) = \mathbf{L}^{\mathrm{T}}(\mathbf{I} - s\mathbf{A})^{-1}\mathbf{Bu}$$

$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots$$

Moments $\mathbf{L}^T\mathbf{R}$ $\quad \mathbf{L}^T\mathbf{AR}$ $\quad \mathbf{L}^T\mathbf{A}^2\mathbf{R}$

Moment matching techniques

Explicit $\quad \mathbf{M}_i = \mathbf{L}^T\mathbf{A}^i\mathbf{R} \;\rightarrow \mathbf{H}_q(s)$

Asymptotic Waveform Evaluation (AWE)

Pade` Approximations

Complex Frequency Hopping (CFH)

- Good theoretical properties, convergence
- Bad numerical properties, intrinsic ill-conditioning due to
  - Moment generation
  - Moment matching

Moment matching techniques

Implicit

Krylov subspace projection methods

- Same information stored in moments
- Much better numerical performance, robustness
- Several versions

  Arnoldi, PRIMA, Lanczos, …
- Possibility of preserving stability and passivity

  by construction!

Krylov subspaces

$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots$$

Moments $\mathbf{L}^T\mathbf{R}$ $\mathbf{L}^T\mathbf{A}\mathbf{R}$ $\mathbf{L}^T\mathbf{A}^2\mathbf{R}$

$$\boxed{Kr(\mathbf{A},\mathbf{R},q) = \mathrm{span}\left\{\mathbf{R},\mathbf{A}\mathbf{R},\mathbf{A}^2\mathbf{R},\ldots,\mathbf{A}^{q-1}\mathbf{R}\right\}}$$

$$\mathbf{V}_q = \text{basis of } Kr(\mathbf{A},\mathbf{R},q)$$

Constructed via iterative (stable) algorithms

Arnoldi (basic) algorithm

$$\begin{cases} \mathbf{x} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{R}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^{T}\mathbf{x} \end{cases}$$

$$\mathbf{V}_q^{T} \; \mathbf{A} \; \mathbf{V}_q = \mathbf{A}_q$$

$$\mathbf{x} \cong \mathbf{V}_q \, \mathbf{x}_q$$

$$\begin{cases} \mathbf{x}_q = \mathbf{A}_q\dot{\mathbf{x}}_q + \mathbf{R}_q\mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^{T}\mathbf{x}_q \end{cases}$$

PRIMA algorithm

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^{T}\mathbf{x} \end{cases}$$

$$\mathbf{V}_q^{T} \; \mathbf{C} \; \mathbf{V}_q = \mathbf{C}_q$$

$$\mathbf{V}_q^{T} \; \mathbf{G} \; \mathbf{V}_q = \mathbf{G}_q$$

$$\mathbf{x} \cong \mathbf{V}_q \, \mathbf{x}_q$$

$$\begin{cases} \mathbf{G}_q\mathbf{x}_q + \mathbf{C}_q\dot{\mathbf{x}}_q = \mathbf{B}_q\mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^{T}\mathbf{x}_q \end{cases}$$

Passivity conditions (PRIMA)

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^{\mathrm{T}}\mathbf{x} \end{cases}$$

Can often be enforced by construction building the original system

$$\begin{cases} \mathbf{G}_q\mathbf{x}_q + \mathbf{C}_q\dot{\mathbf{x}}_q = \mathbf{B}_q\mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^{T}\mathbf{x}_q \end{cases}$$

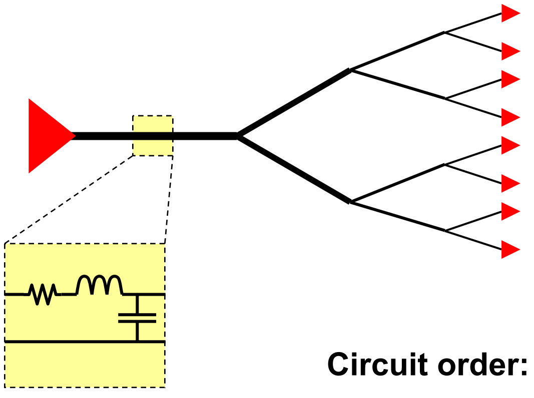
$\mathbf{G} \geq 0 \qquad \mathbf{C} \geq 0$

$\mathbf{C}$ symmetric

$\mathbf{L} = \pm\mathbf{B}$

$\mathbf{V}_q$ must be full rank

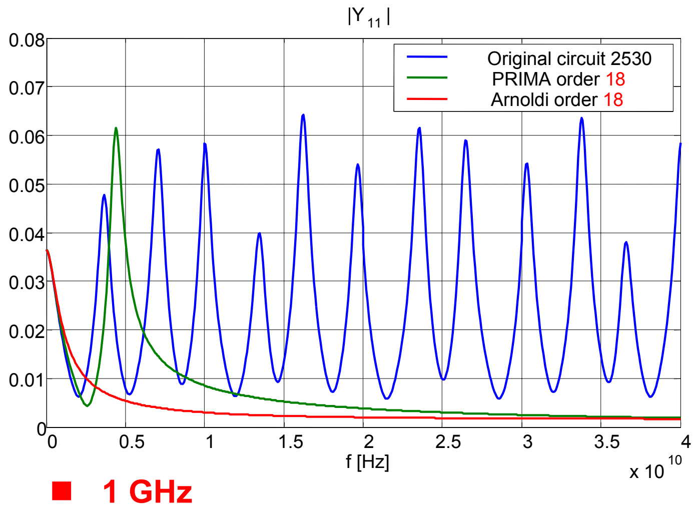
An example: RLC tree circuit



Circuit order: 2530

Ports: 9

RLC tree circuit: order reduction



■ 1 GHz

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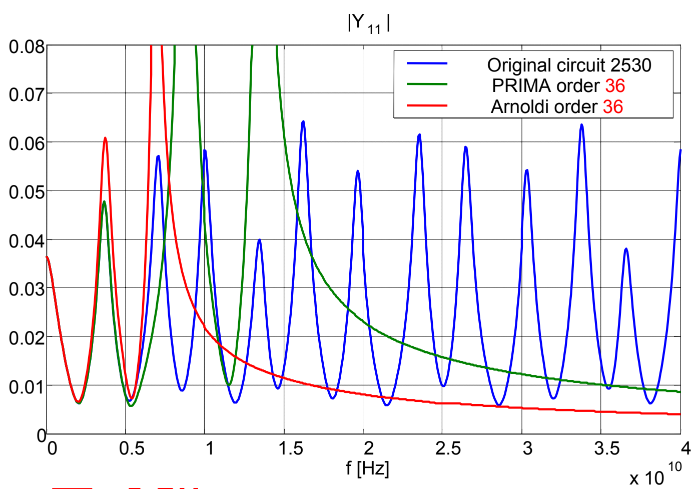
RLC tree circuit: order reduction



■ 2 GHz

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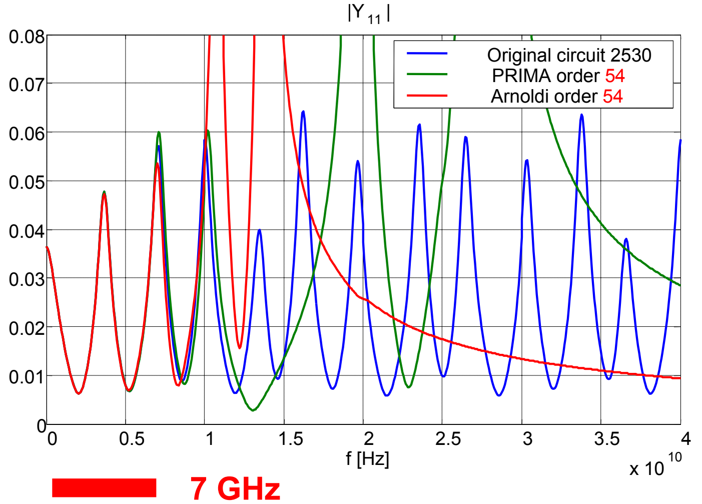
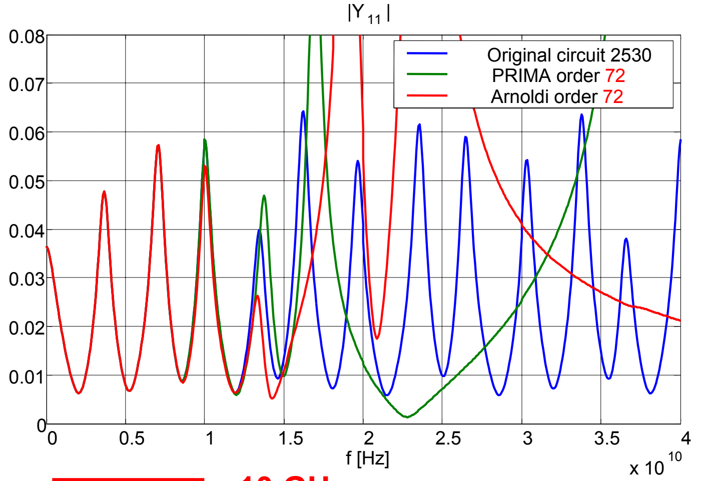
RLC tree circuit: order reduction



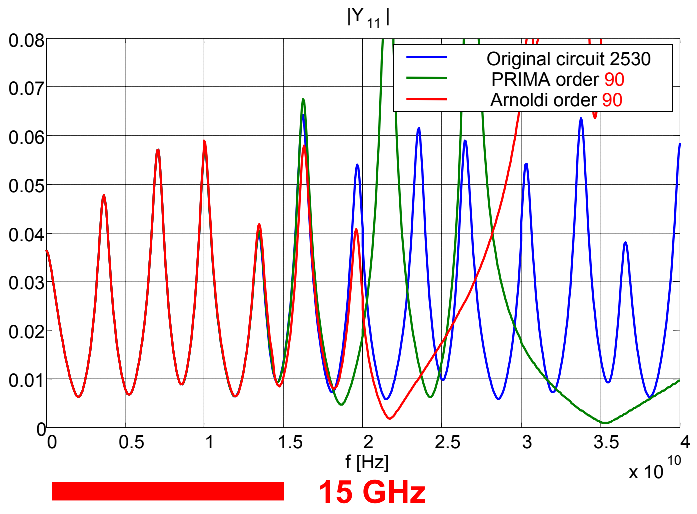
7 GHz

RLC tree circuit: order reduction



10 GHz

RLC tree circuit: order reduction



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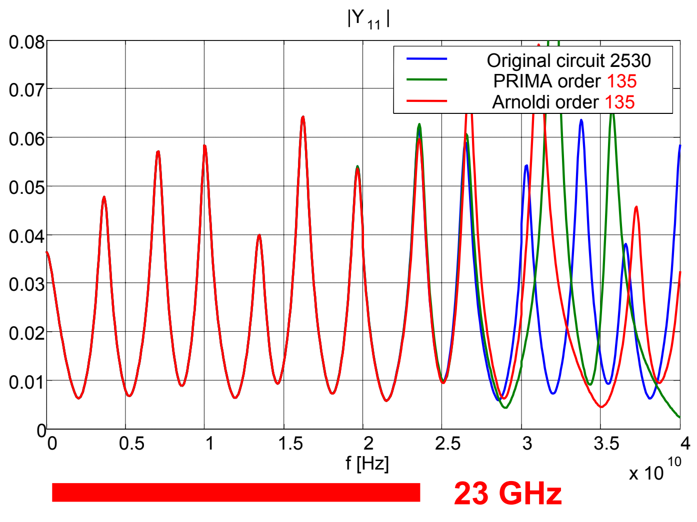
RLC tree circuit: order reduction



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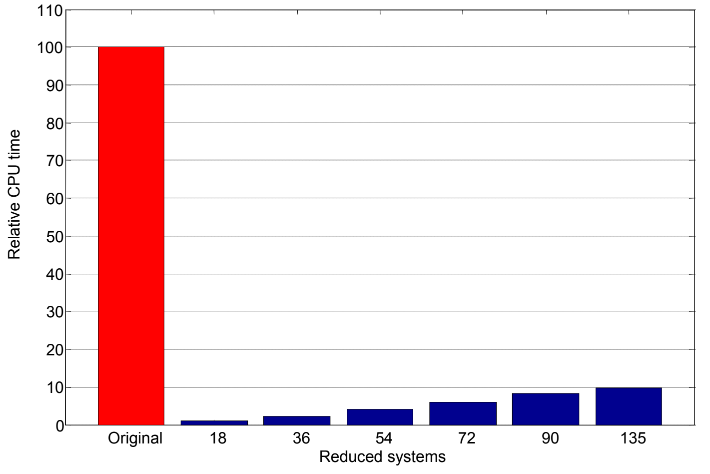
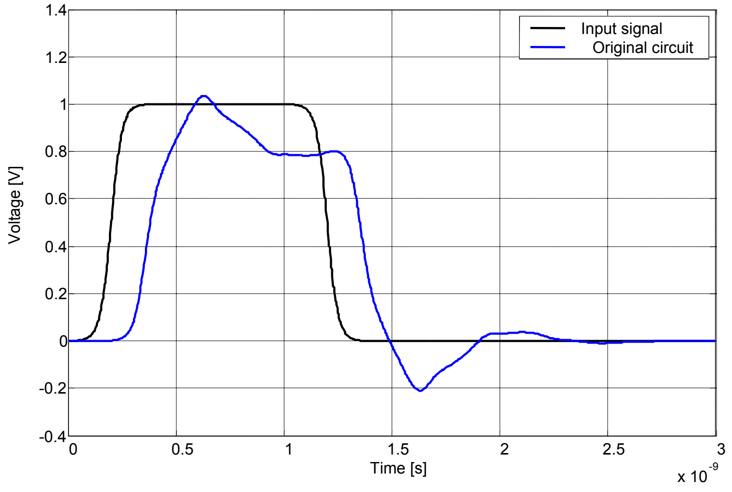
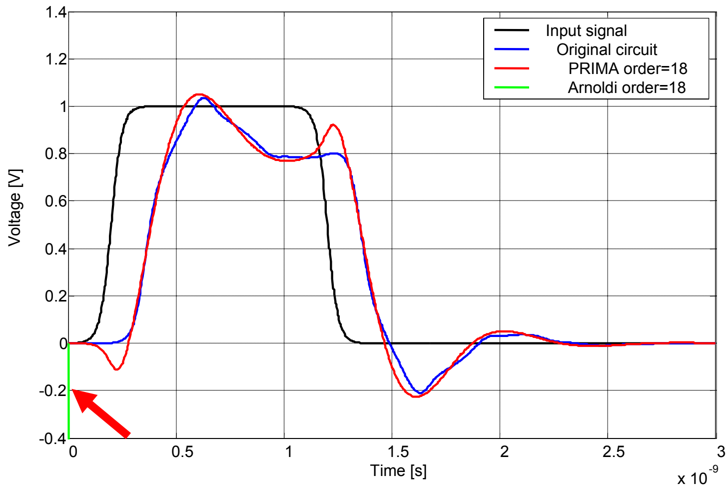
RLC tree circuit: efficiency



RLC tree circuit: transient analysis

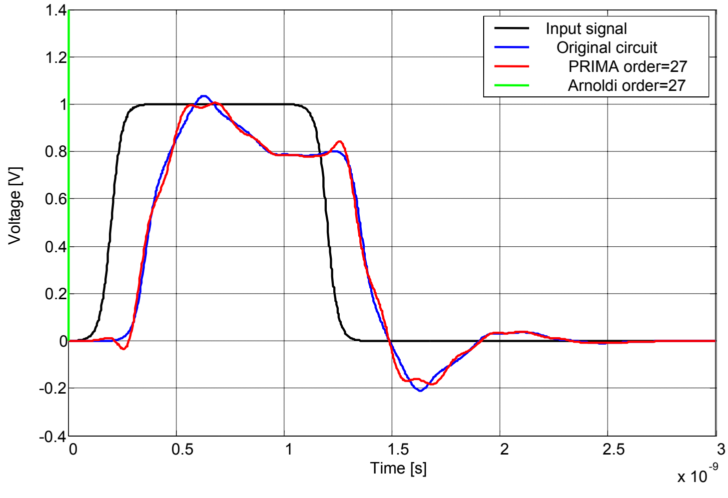


RLC tree circuit: transient analysis



S. Grivet-Talocia, SPI tutorial, 9 May 2004 **43**

RLC tree circuit: transient analysis



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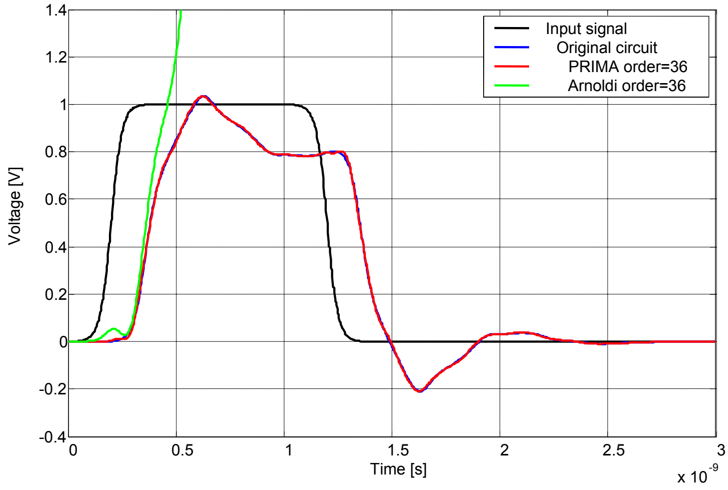
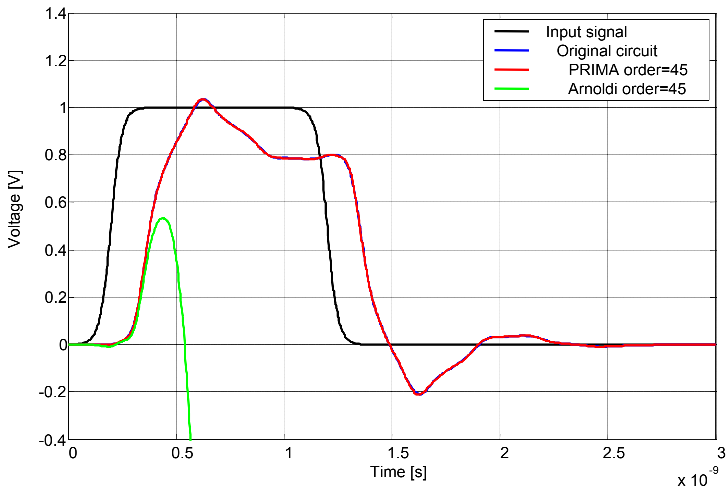
RLC tree circuit: transient analysis

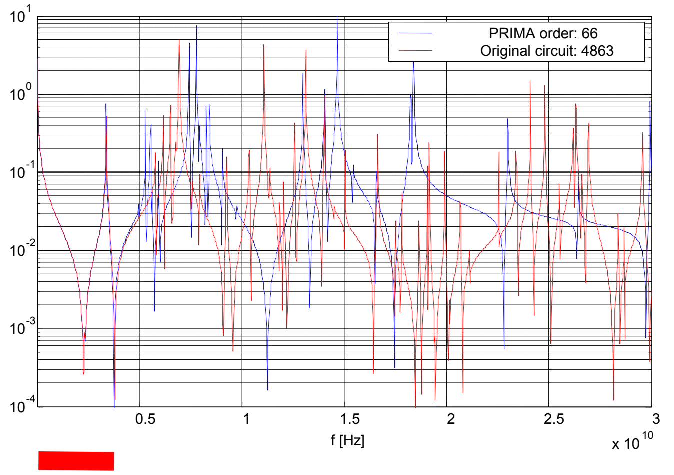


RLC tree circuit: transient analysis



Example: MNA, 22 ports, order 4863

(www.win.tue.nl/niconet/NIC2/benchmodred.html)



3 GHz

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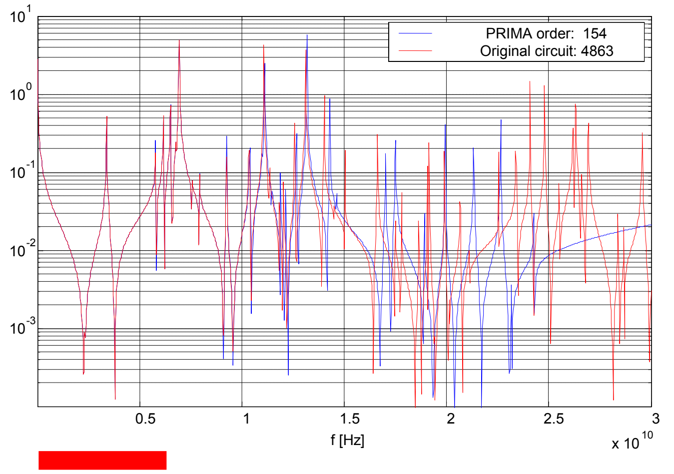
Example: MNA, 22 ports, order 4863



6 GHz

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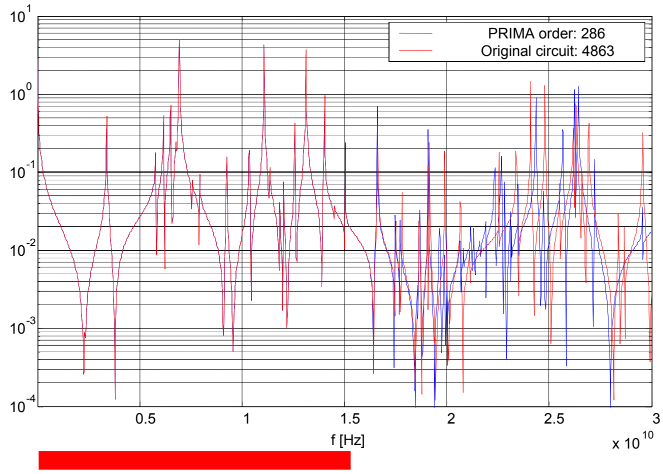
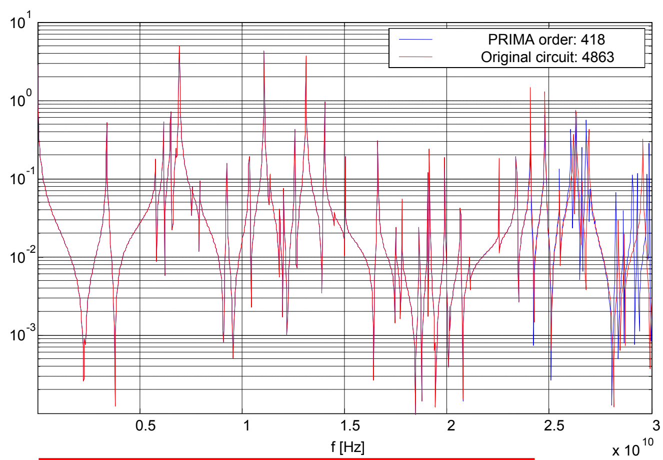
Example: MNA, 22 ports, order 4863



15 GHz

Example: MNA, 22 ports, order 4863



24 GHz

Key references

M.Celik, L.Pileggi, A.Odabasioglu, IC Interconnects Analysis, Kluwer, 2002

…and references therein

R.Achar, M.S.Nakhla, Proceedings of the IEEE, Vol.89, 2001, 693-728

… and references therein

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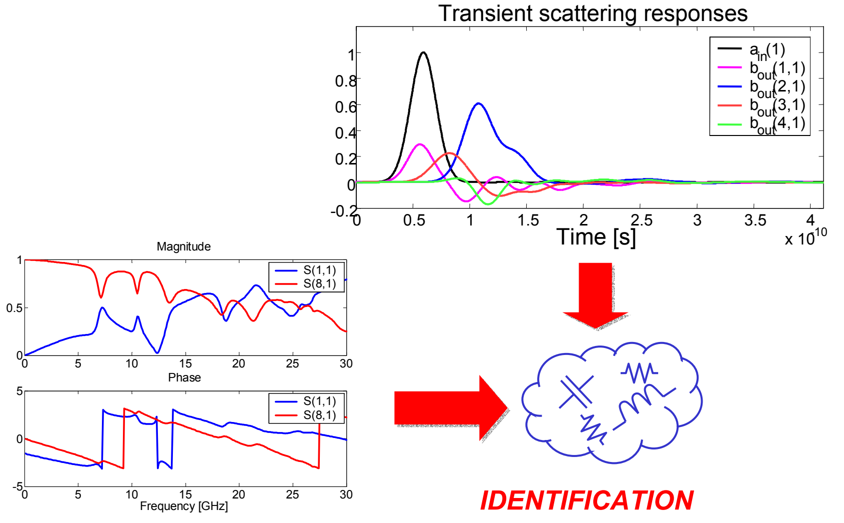
- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- $\boxed{\text{Model Identification methods}}$
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

Model Identification



Model identification

From samples to model: identification process

Reduced-order identification: approximation process

Several identification methods exist

Characterized by use of different:

- Input data
- Modeling criteria
- Model parameter estimation

Identification methods

Block Complex Frequency Hopping (BCFH)

[R.Achar, M.S.Nakhla, IEEE Proceedings, Vol.89, 2001]

- Rational Padé approximation of network functions
- Convergence property in a neighborhood of the expansion point
- Hopping along frequency axis to cover the modeling bandwidth
- May lead to ill-conditioned numerical systems when used for identification from sampled responses

Identification methods

Global Rational Approximation

[M.Elzinga, K.L.Virga, J.L.Prince, IEEE Trans. MTT, 9/2000]

[W.Beyene, J.Schutt-Aine, IEEE Trans. CPMT, Vol.21, 3/1998]

[K.L.Choi, M.Swaminathan, IEEE Trans. CAS II, vol.47, 4/2000]

[J.Morsey, A.C.Cangellaris, Proc. EPEP, 2001]

[… many, many, many others…]

- A matrix of rational functions is fitted to the samples of a network function matrix (e.g. the Y matrix)

Identification methods

Pencil of Functions

[Y.Hua, T.Sarkar, IEEE Trans. AP, vol.37, 2/1989]

- Time-domain data
- Estimates model poles by fitting a sum of exponential functions to the samples of transient port responses
- Poles obtained as eigenvalues of a generalized eigenvalue problem
- Automatic order estimation

Identification methods

Subspace-based State-Space System Identification methods (4SID)

[M. Viberg, Automatica, 12/1995]

[T.McKelvey, H.Akcay, L.Ljung, IEEE Trans. AC, vol.41, 7/1996]

- Based on projections of data onto orthogonal subspaces, leading to direct state-space estimation
- Built-in automatic order estimation (based on SVD)
- Available in both time and frequency domain
- Equivalent to Pencil of Functions methods

Identification methods

Nevanlinna-Pick Interpolation

[C.P.Coelho, J.R.Phillips, L.M.Silveira, Proc. DATE 2002]

- Interpolation of samples of the scattering matrix with a (unitary bounded) matrix rational function
- Nice theoretical properties
- Very complex
- Leads to models with large dynamical order

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Identification methods

Vector Fitting

[B.Gustavsen, A. Semlyen, IEEE Trans. PD, vol.14, 7/1999]

- Performs data fitting with rational functions avoiding nonlinear optimization
- Iterative process converging to the dominant poles
- Available for both time and frequency domain

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Identification methods

Identification methods are not expected to work for every possible problem

Any method performs well for a certain class of identification problems

Vector Fitting is selected here as one of the most promising methods for a wide range of applications

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Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

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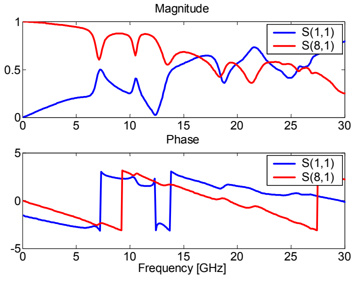
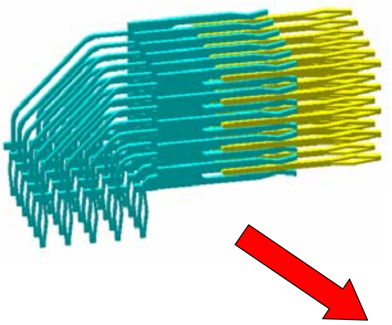
Frequency-domain macromodeling

Model identification from frequency-domain responses

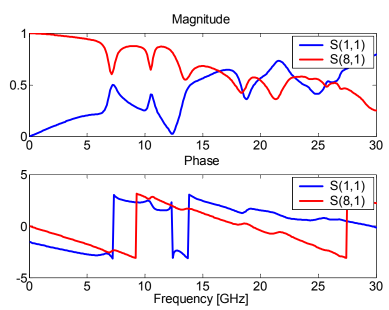


IDENTIFICATION

Possible scenarios

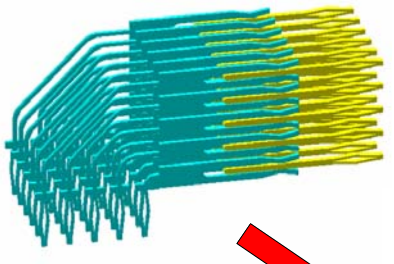


Frequency-Domain
full-wave simulation
(MoM, FEM, …)



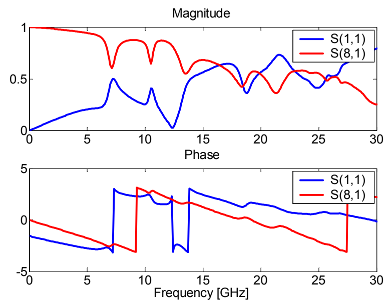
Frequency tables
of transfer matrix
(S, Y, Z, …)

Possible scenarios



Time-Domain full-wave simulation (FIT, FDTD)

FFT postprocessing



Frequency tables of transfer matrix (S, Y, Z, …)

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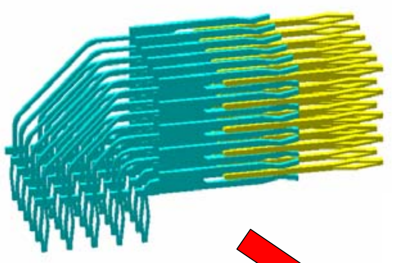
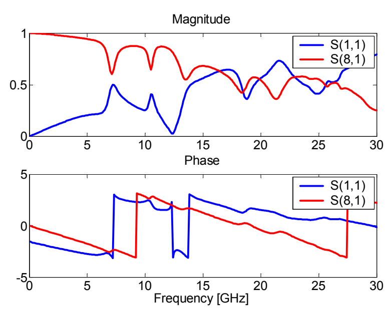
Possible scenarios



Direct VNA measurement



Frequency tables of transfer matrix (S)

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Frequency-Domain Macromodeling

Input data

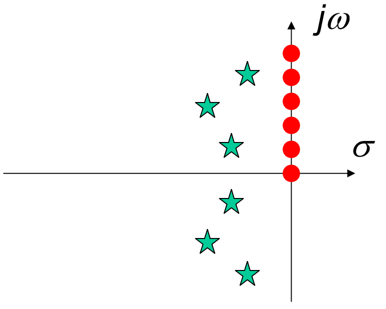
$$\left\{ \hat{H}(j\omega_k), \quad k = 1, \dots, K \right\}$$

Approximation

$$H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty$$

Fitting condition

$$H(j\omega_k) \approx \hat{H}(j\omega_k), \forall k$$



Unknowns:
- Poles p_n
- Residues R_n
- Constant H_∞

Frequency-Domain Macromodeling

Direct fitting condition: nonlinear!

$$\sum_{n=1}^{N} \frac{R_n}{j\omega_k - p_n} + H_\infty \approx \hat{H}(j\omega_k), \quad \forall k$$

- Nonlinear dependence on poles
- Requires nonlinear optimization (e.g. nonlinear least squares)
- Convergence problems (local minima, etc…)

Frequency-Domain Macromodeling

Direct fitting condition: nonlinear!

$$\sum_{n=1}^{N} \frac{R_n}{j\omega_k - p_n} + H_\infty \approx \hat{H}(j\omega_k), \quad \forall k$$

Vector Fitting avoids nonlinear optimization

B. Gustavsen, A. Semlyen, ``Rational approximation of frequency responses by **vector fitting**'', *IEEE Trans. Power Delivery*, Vol.14, July 1999, pp.1052-1061

Frequency-Domain Vector Fitting

Input data

$$\left\{ \hat{H}(j\omega_k), \quad k = 1,\ldots,K \right\}$$

Approximation

$$H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty$$

Weight function

$$w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1$$

- $w(s)$ is unitary for $s \to \infty$
- poles q_n are fixed a priori
- residues c_n are unknown

Vector Fitting condition

$$w(s)\, H(s) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$

The poles of $w(s)\, H(s)$ are $\{ q_n \}$ only!

Frequency-Domain Vector Fitting

Input data

$$\left\{ \hat{H}(j\omega_k), \quad k = 1,\dots,K \right\}$$

Approximation

$$H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty$$

Weight function

$$w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1 = \prod_{n=1}^{N} \frac{(s - z_n)}{(s - q_n)}$$

There are N zeros $\{ z_n \}$

Vector Fitting condition

$$w(s)\, H(s) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$

$$\{p_n\} \cong \{z_n\}$$

Frequency-Domain Vector Fitting

Input data

$$\left\{ \hat{H}(j\omega_k), \quad k = 1,\dots,K \right\}$$

Weight function

$$w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1$$

$$\left\{ \sum_{n=1}^{N} \frac{c_n}{j\omega_k - q_n} + 1 \right\} \hat{H}(j\omega_k) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{j\omega_k - q_n} + \tilde{H}_\infty$$

Vector Fitting condition

$$w(s)\, H(s) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$

Frequency-Domain Vector Fitting

Unknowns

$\{c_n, \tilde{c}_n, \tilde{H}_\infty\}$

$$\left\{\sum_{n=1}^{N} \frac{c_n}{j\omega_k - q_n} + 1\right\} \hat{H}(j\omega_k) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{j\omega_k - q_n} + \tilde{H}_\infty$$

Linear least squares problem: easy to solve!

$$w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1 = \prod_{n=1}^{N} \frac{(s - z_n)}{(s - q_n)}$$

Poles of $H(s)$

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Frequency-Domain Vector Fitting

$$w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1 = \prod_{n=1}^{N} \frac{(s - z_n)}{(s - q_n)}$$

Poles of $H(s)$

Theorem: the zeros $\{ z_n \}$ are the eigenvalues of

$$\mathbf{Q} = \mathbf{A} - \mathbf{b}\,\mathbf{c}^T$$

where

$$\mathbf{A} = \operatorname{diag}\{ q_n \}$$

$$\mathbf{b} = (1 \quad 1 \quad \cdots \quad 1)^T$$

$$\mathbf{c} = (c_1 \quad c_2 \quad \cdots \quad c_N)^T$$

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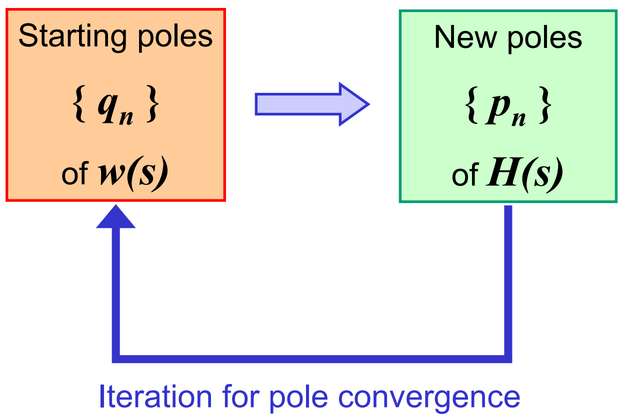
Pole relocation



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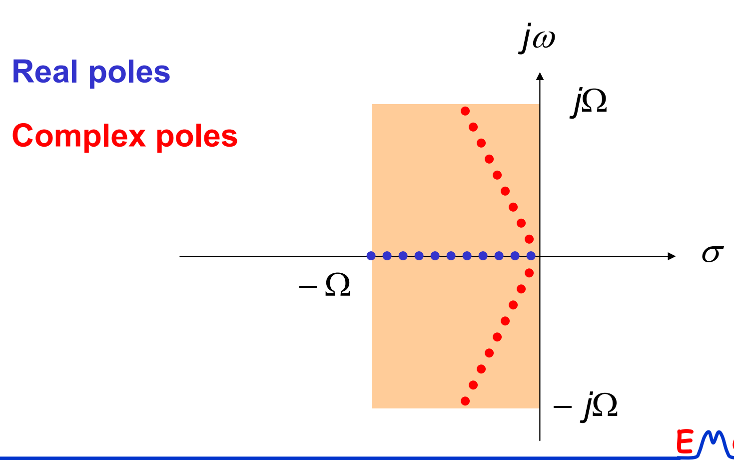
Choice of starting poles

Uniform coverage of approximation bandwidth



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Vector Fitting: residues

Input data

$$\left\{ \hat{H}(j\omega_k), \quad k = 1, \ldots, K \right\}$$

Approximation

$$H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty$$

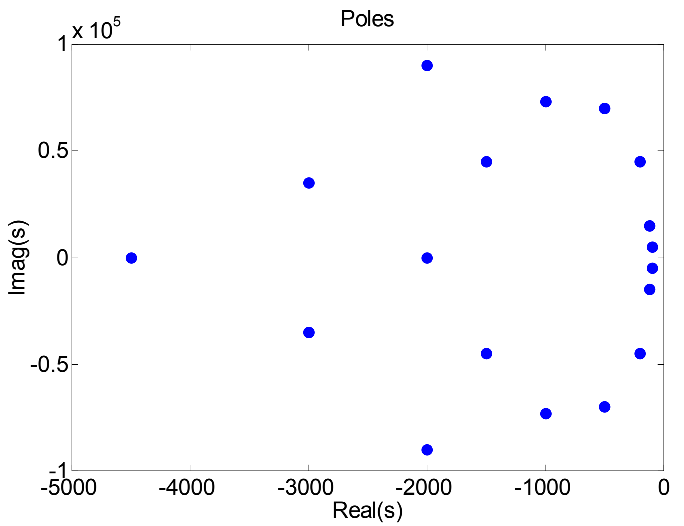
Fitting condition

$$H(j\omega_k) \approx \hat{H}(j\omega_k), \forall k$$

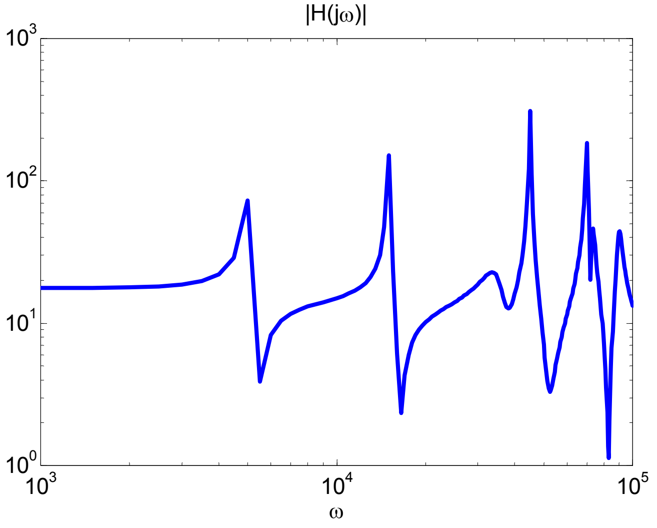
Another linear least squares problem!

Residues R_n and constant H_∞

Example 1: 18 random poles/residues



Example 1



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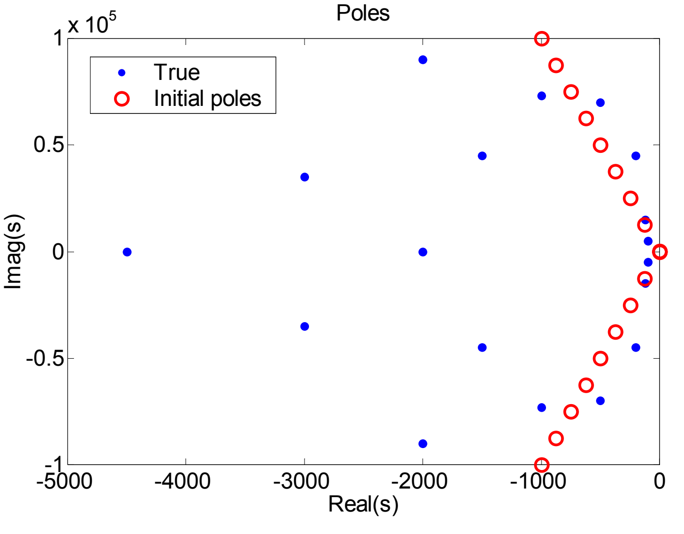
Example 1



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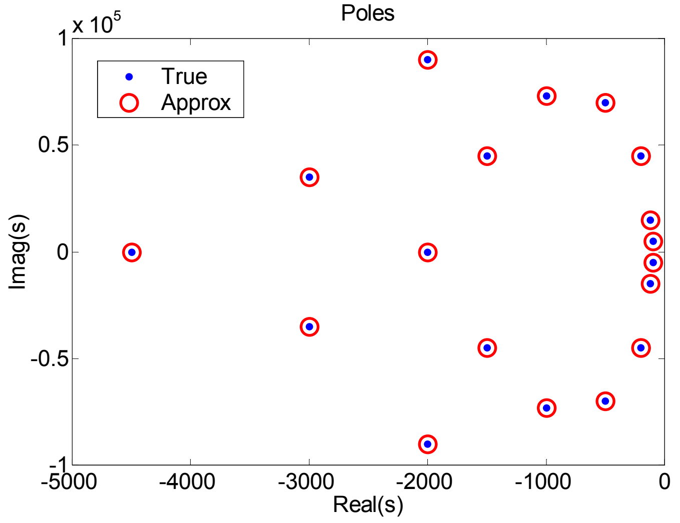
Example 1



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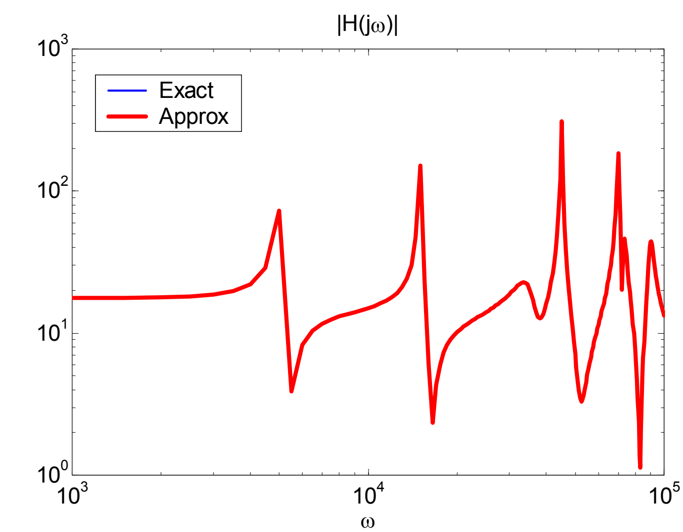
Example 1



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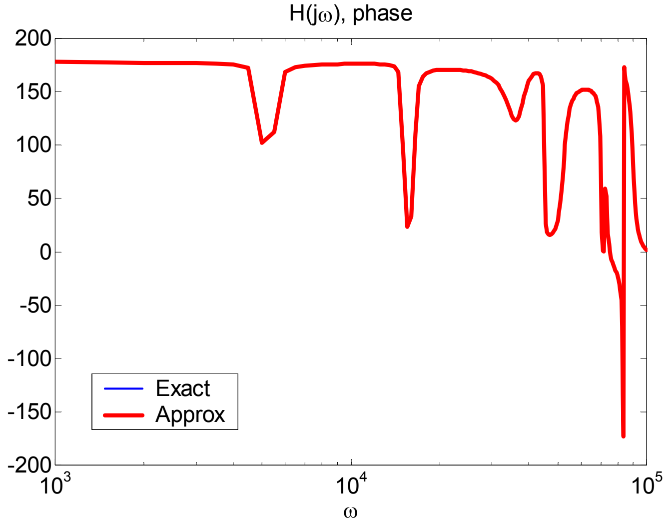
Example 1

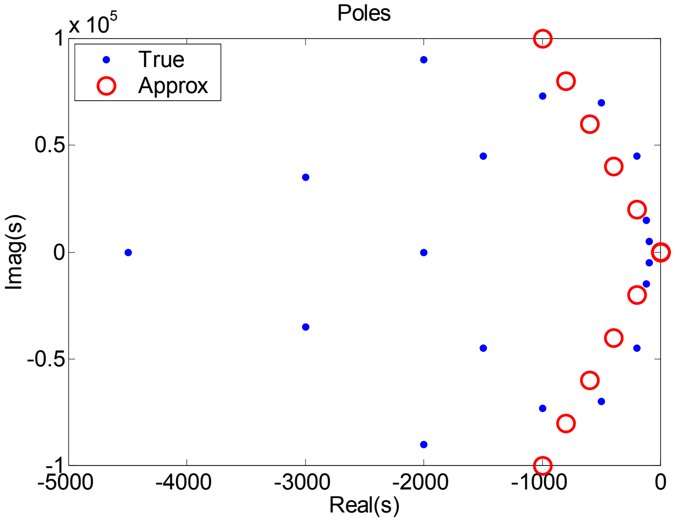


Example 2

Same 18-pole rational function

Reduced-order fitting (12th order)

Example 2



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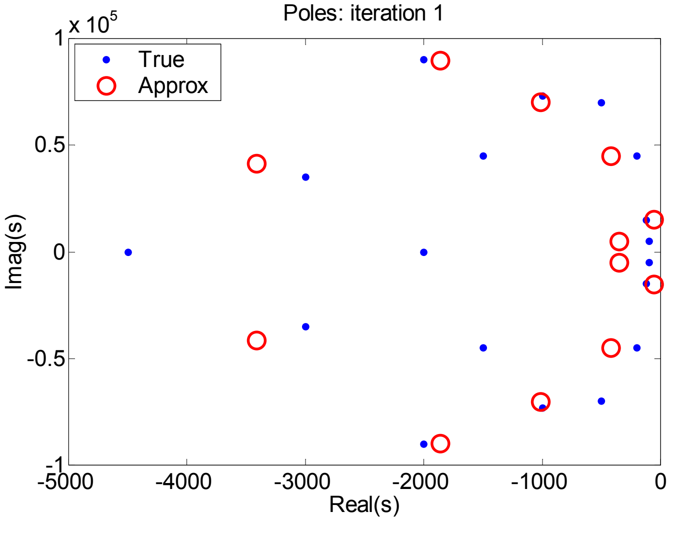
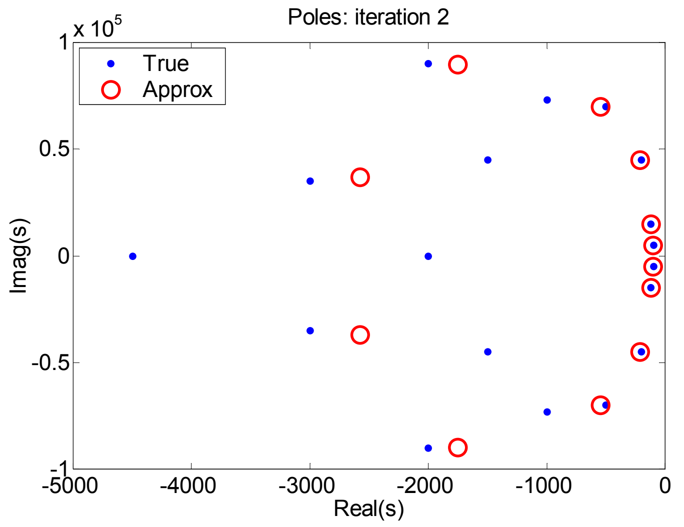
Example 2



EMC GROUP

Example 2



EMC GROUP

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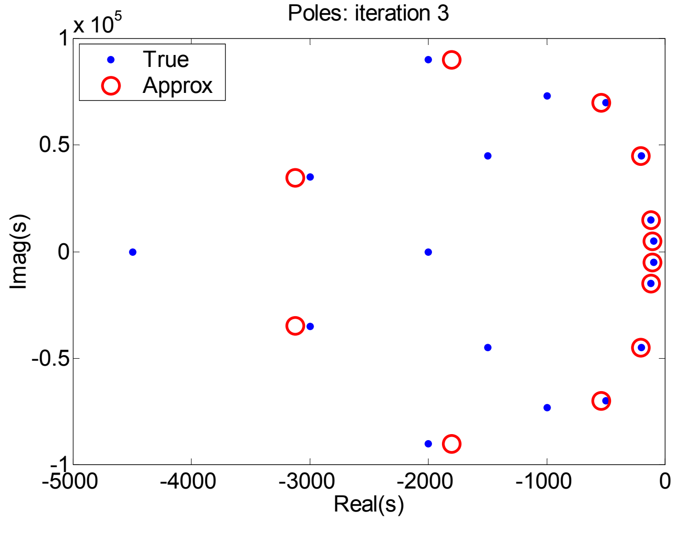

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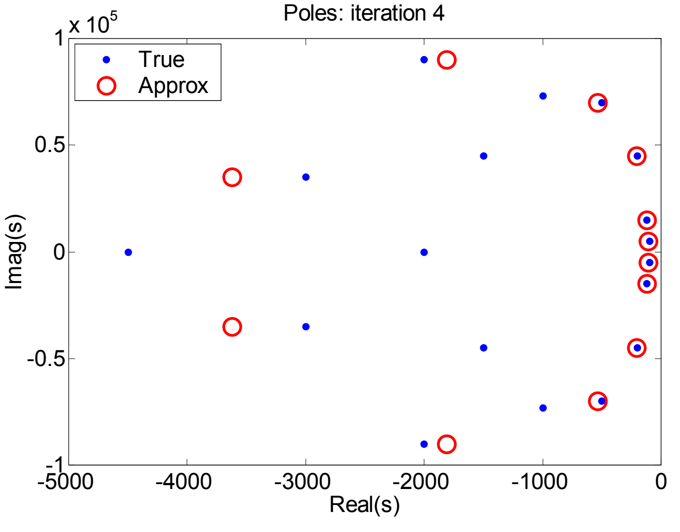

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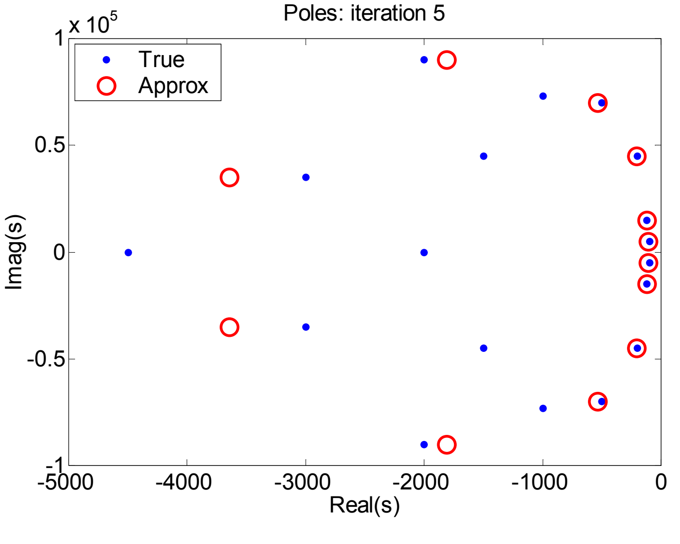
Example 2



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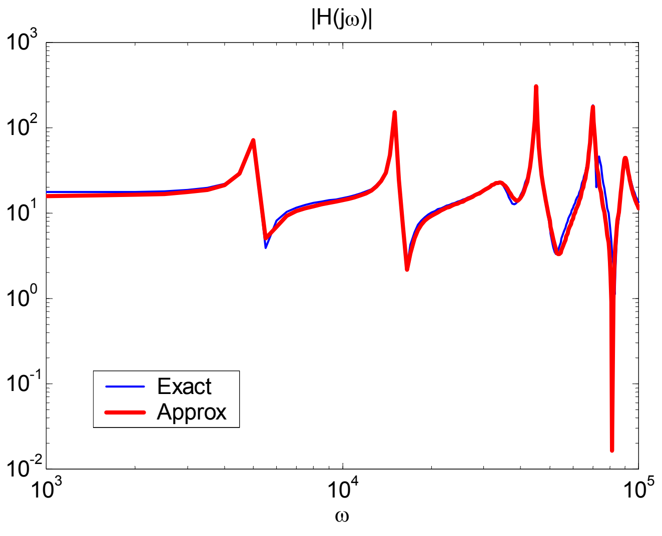
Example 2



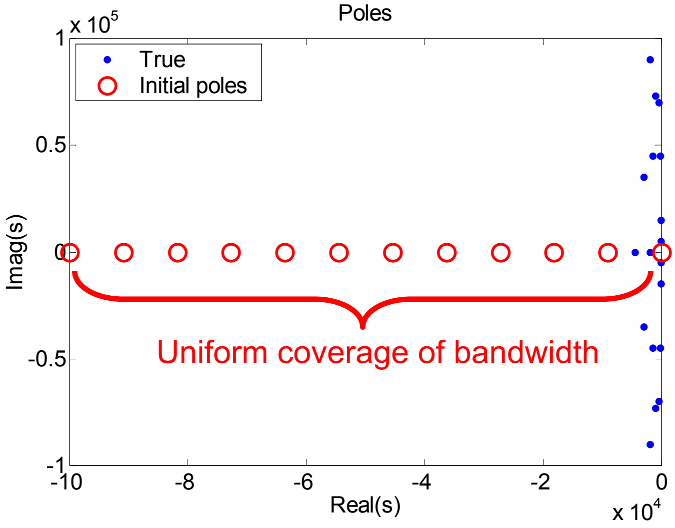
Example 3

Same 18-pole rational function

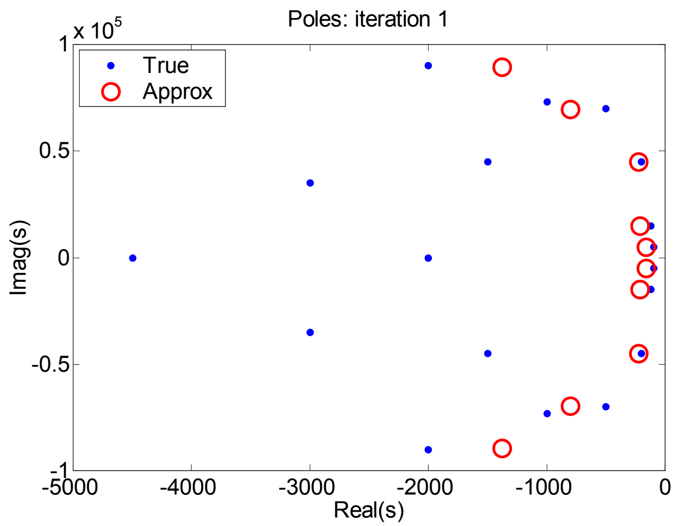
Reduced-order fitting (12th order)

Different starting poles (real poles)

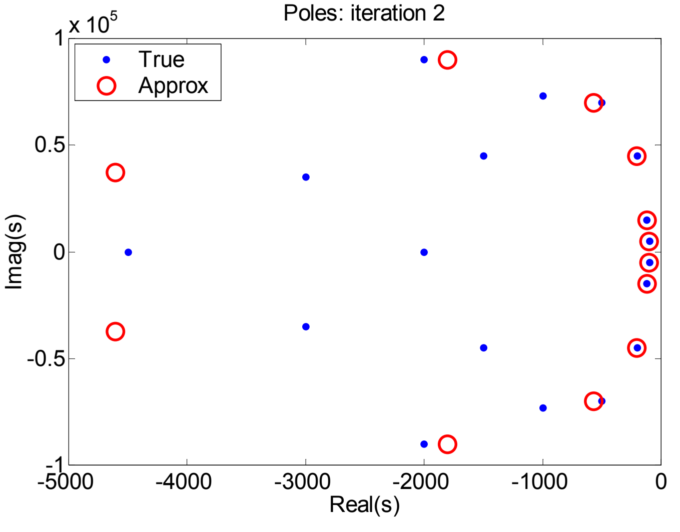
Example 3



Example 3



Example 3



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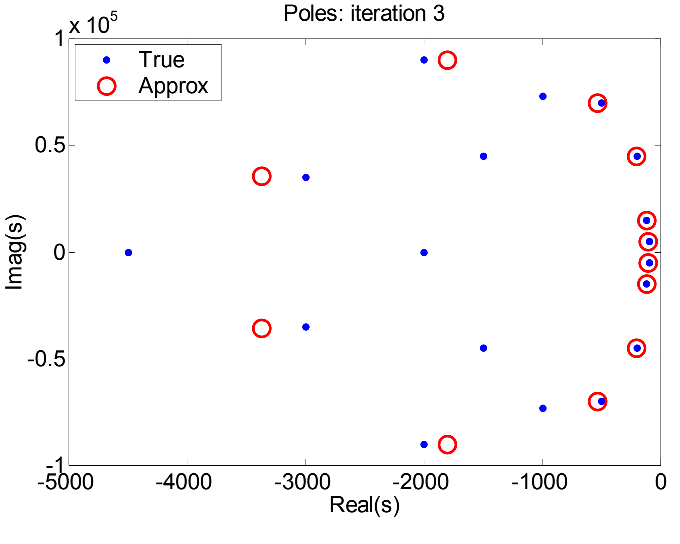
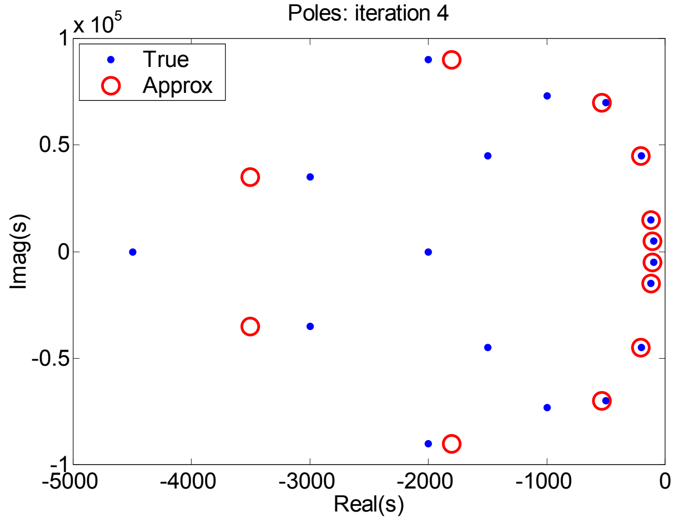


Example 3



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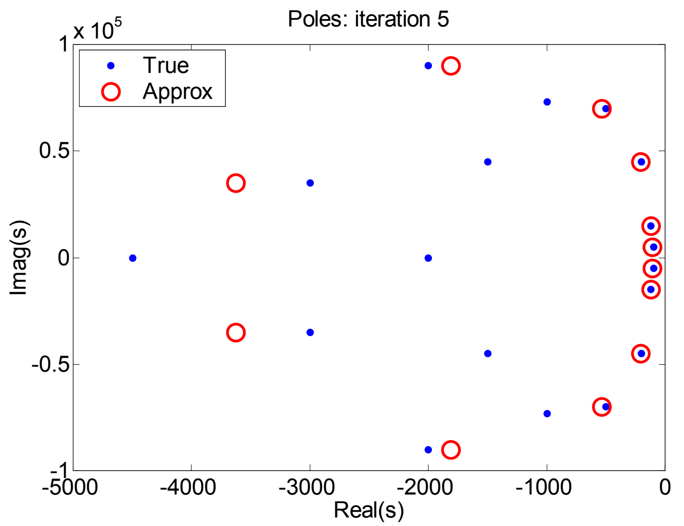
Example 3



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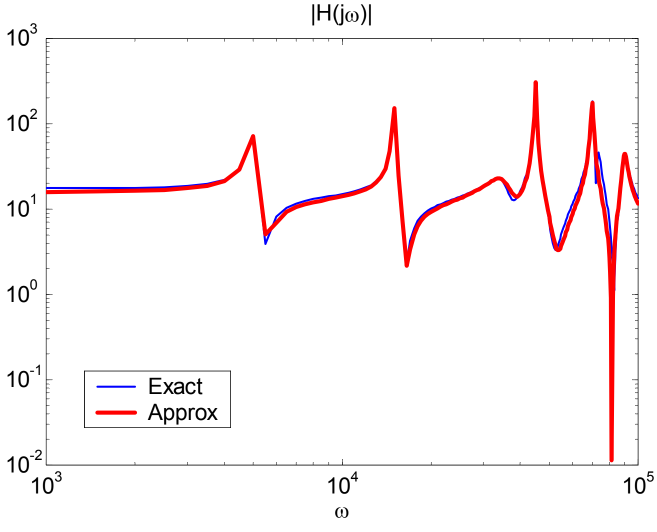
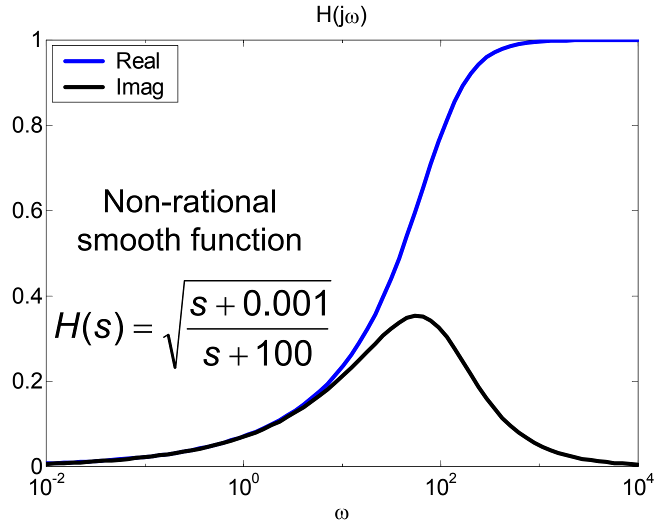
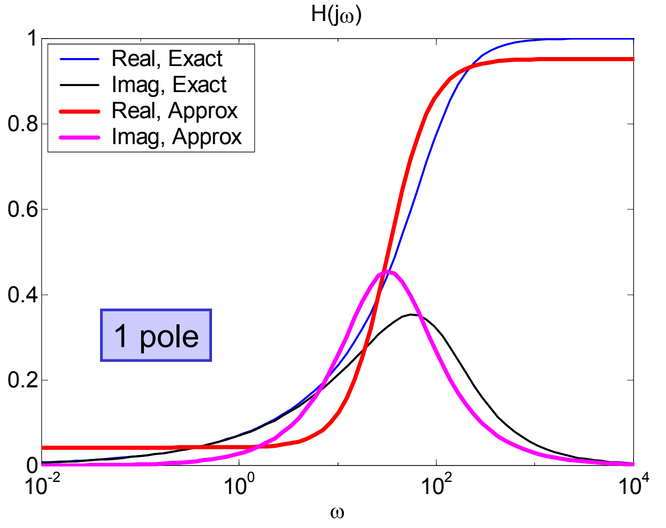
Example 3



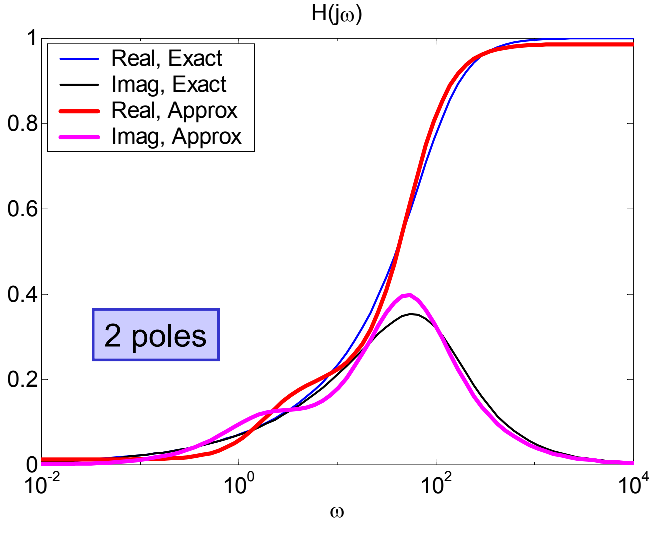
Example 4



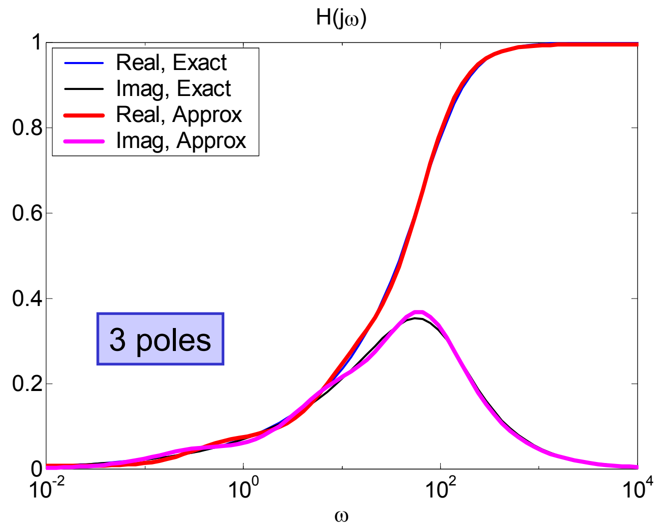
Example 4



Example 4



Example 4



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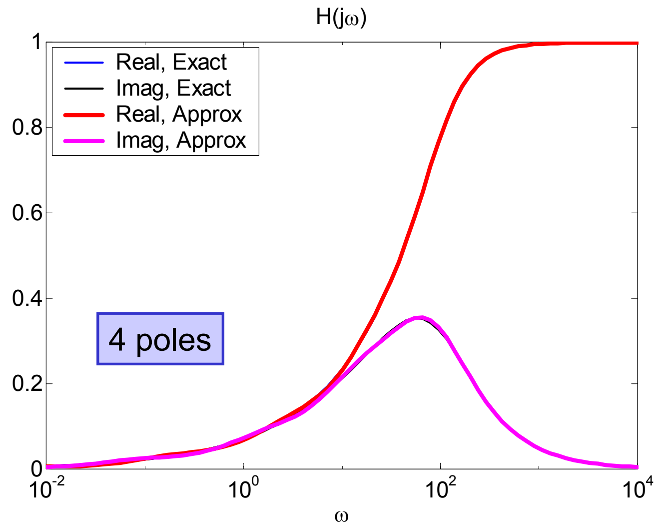
Example 4



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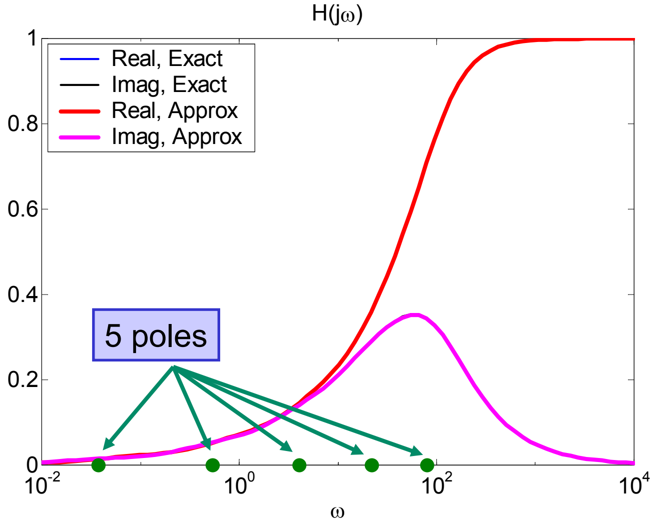
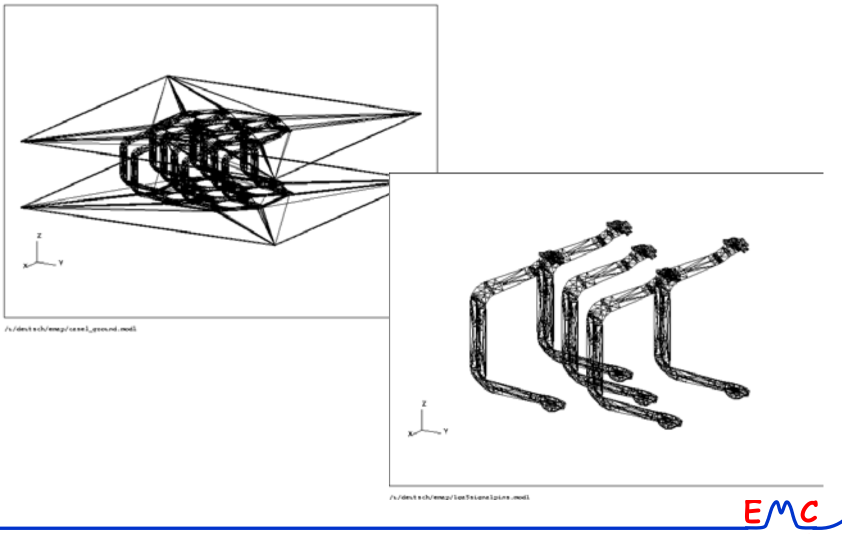
Example 4

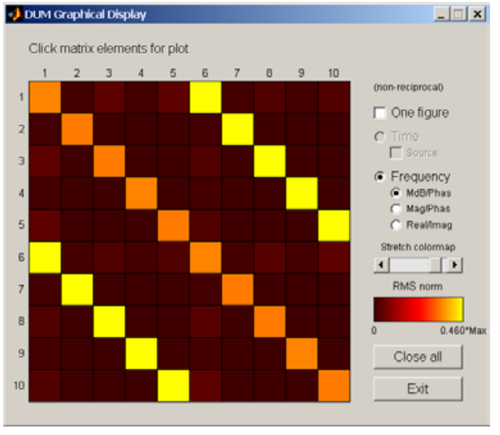


Example 5: MCM-board connector

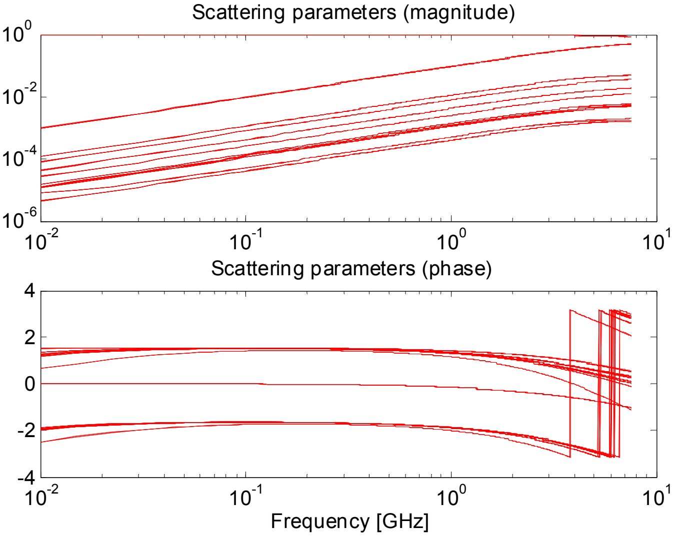


Example 5: MCM-board connector

Data: 10-port structure, frequency-domain S-matrix

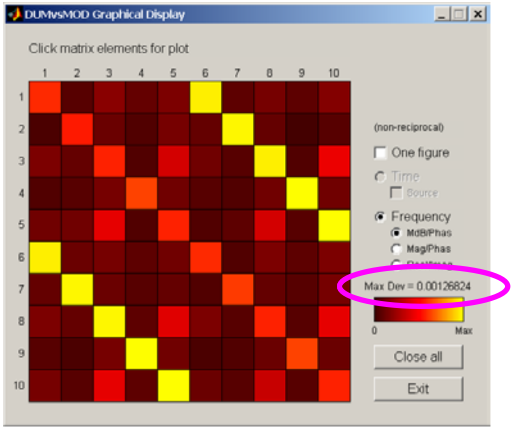


Example 5: MCM-board connector



Example 5: MCM-board connector

Macromodel: 4-poles



Error: 0.1%

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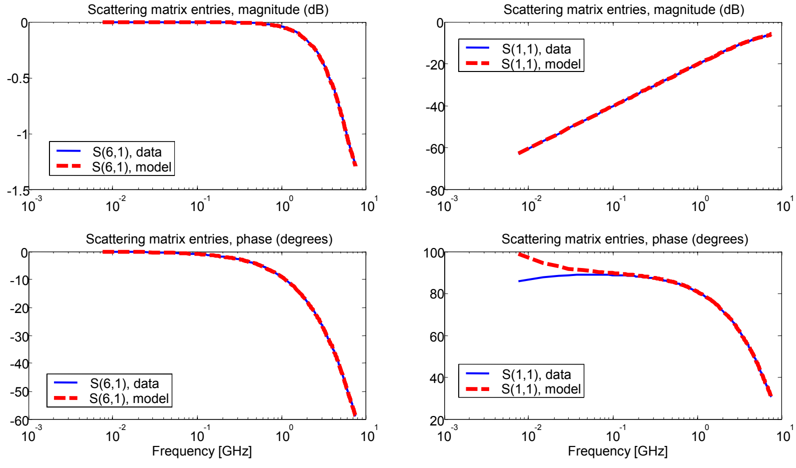
Example 5: MCM-board connector



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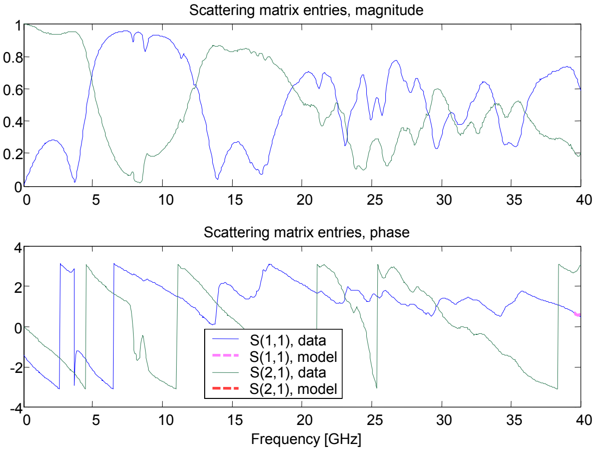
Example 6: stripline+lauches

Data: measured S-parameters



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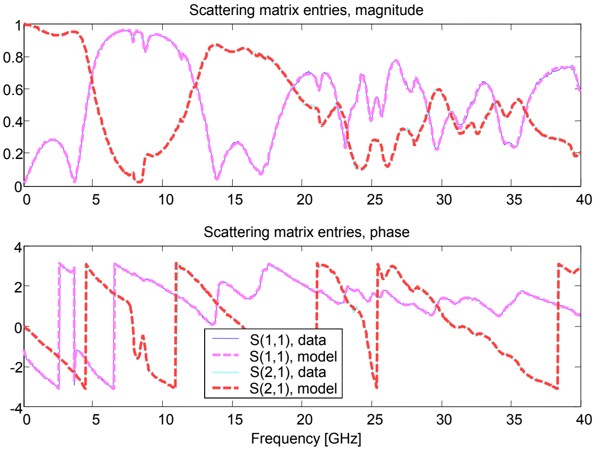
Example 6: stripline+lauches

Macromodel: 60 poles



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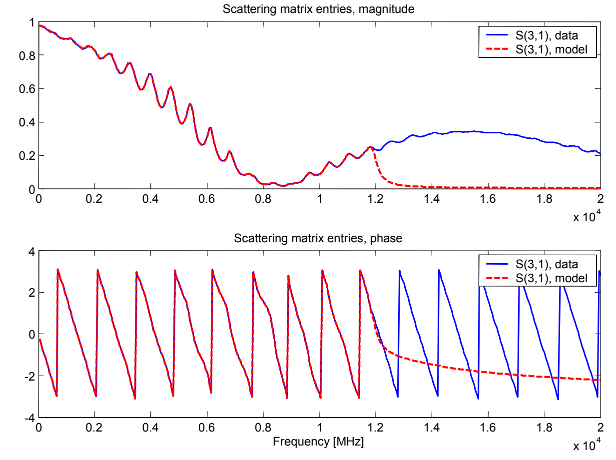
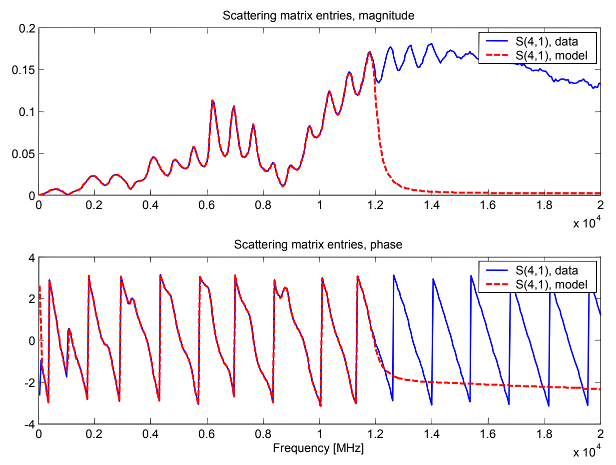
Example 7: PCB path, measured

VF with frequency-selective weighting



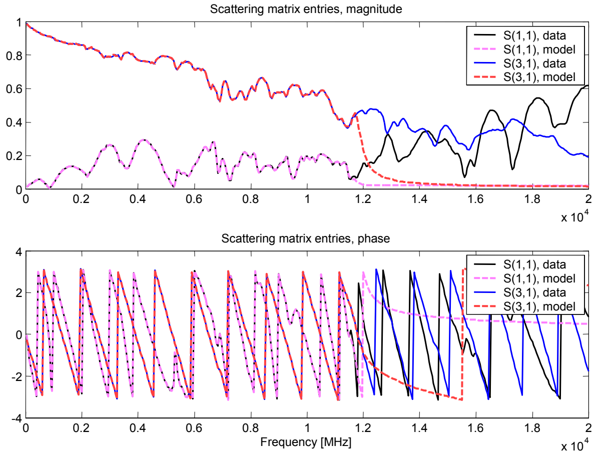
Example 7: PCB path, measured

VF with frequency-selective weighting



Example 8: Connector, measured

VF with frequency-selective weighting



Vector Fitting: summary

- Tool for frequency-domain rational approximation
  - rational transfer functions (system identification)
  - rational transfer functions (reduced-order modeling)
  - non-rational transfer functions
- **Data from full-wave simulations**
- **Direct frequency-domain measurements**

Vector Fitting: summary

`www.energy.sintef.no/produkt/VECTFIT/home.asp`

- Very accurate and robust
- Only linear least squares + eigenvalues required
- Stability is not guaranteed
  - can be fixed by flipping real part during relocation
- Passivity is not guaranteed
  - can be fixed a posteriori (see later)

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  - Time-Domain Vector Fitting
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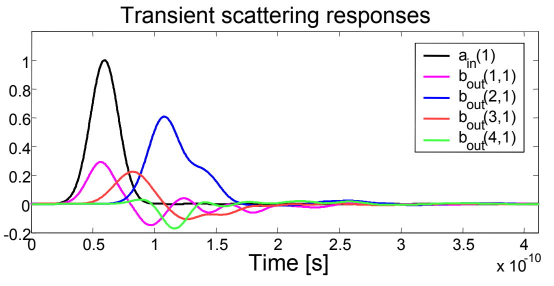
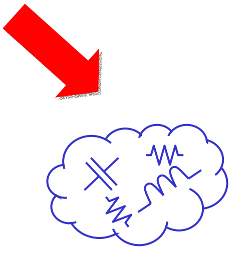
- SPICE synthesis

Time-domain macromodeling

Model identification from time-domain responses



IDENTIFICATION



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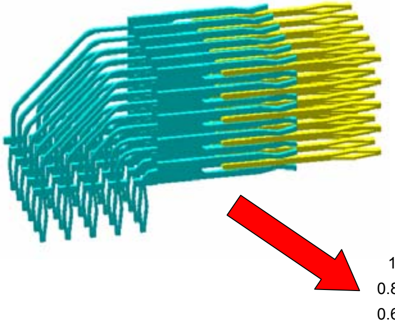
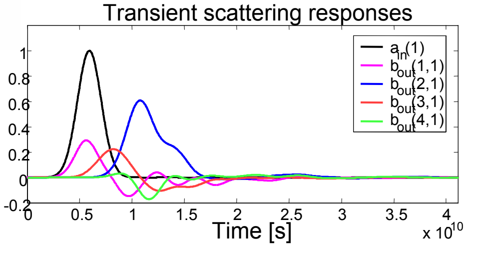
Possible scenarios



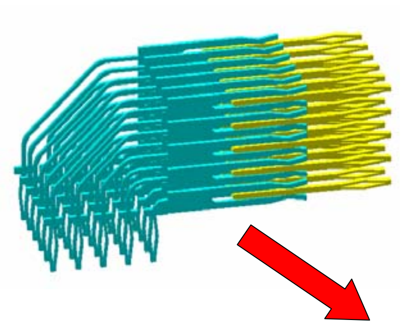
Time-Domain full-wave simulation (FIT, FDTD)

Port responses to transient excitations (usually gaussian)



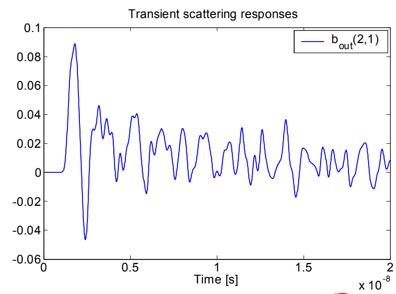
EMC GROUP

Possible scenarios



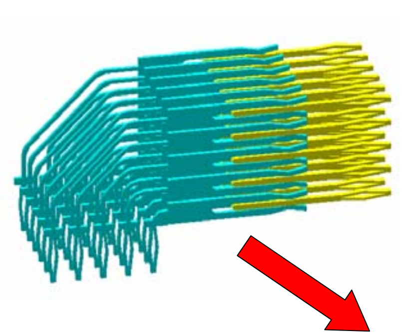
Time-Domain full-wave simulation (FIT, FDTD)

Truncated waveforms from short FDTD runs



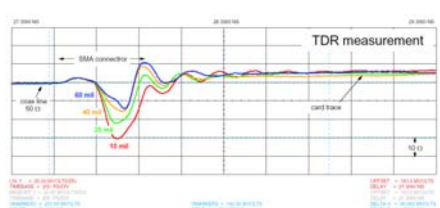
Port responses to transient excitations (usually gaussian)

Possible scenarios



Time-domain measurements

(work in progress)



Port responses to transient excitations (usually gaussian)

Time-Domain Macromodeling

Input pulse

$\mathbf{x}(t)$ t – domain

Output responses

$\mathbf{y}(t)$ t – domain

Transfer function

$$\mathbf{Y}(s) = \mathbf{H}(s)\mathbf{X}(s)$$

Rational approximation

$$\mathbf{H}(s) \approx \mathbf{H}_\infty + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

Unknowns:

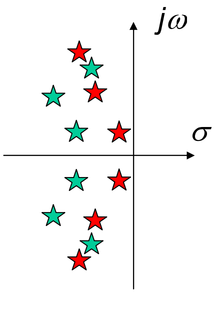
- Poles p_n
- Residues $\mathbf{R}_n$
- Constant $\mathbf{H}_\infty$

Time-Domain Vector Fitting

Step 1. Find the dominant poles via “relocation”

Guess poles $\{ q_n \}$ → New poles $\{ p_n \}$

Iterative refinement



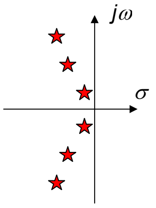
How to do it using time-domain data?

How to insure convergence to the right poles?

Time-Domain Vector Fitting

1a. Start with initial poles: $\{ q_n \}$

1b. Define weight function: unknown $\{ k_n \}$



$$w(s) = 1 + \sum_n \frac{k_n}{s - \boxed{q_n}}$$

Starting poles

1c. Assume the following condition

$$w(s)\mathbf{H}(s) = a + \sum_n \frac{b_n}{s - \boxed{q_n}}$$

Poles of $\mathbf{H}(s)$ = Zeros of $w(s)$

Time-Domain Vector Fitting

$$w(s)\mathbf{H}(s) = a + \sum_n \frac{b_n}{s - q_n}$$

Apply the input pulse $\mathbf{X}(s)$

$$w(s)\mathbf{Y}(s) = \left(a + \sum_n \frac{b_n}{s - q_n} \right) \mathbf{X}(s)$$

Compute inverse Laplace transform

$$\mathbf{y}(t) + \sum_n k_n \mathbf{y}_n(t) = a\,\mathbf{x}(t) + \sum_n b_n \mathbf{x}_n(t)$$

$$\mathbf{x}_n(t) = \int_0^t e^{q_n(t-\tau)} \mathbf{x}(\tau) d\tau \qquad \mathbf{y}_n(t) = \int_0^t e^{q_n(t-\tau)} \mathbf{y}(\tau) d\tau$$

Low-pass filtered input and output signals

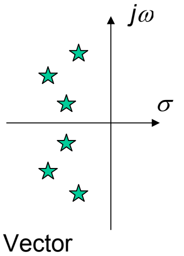
Time-Domain Vector Fitting

1d. Solve a linear least squares system for $\boxed{k_n}$, a , b_n

$$\boxed{\mathbf{y}(t) + \sum_n k_n\, \mathbf{y}_n(t) = a\, \mathbf{x}(t) + \sum_n b_n\, \mathbf{x}_n(t)}$$

1e. Compute the zeros $\{ p_n \}$ of the weight function

$$w(s) = 1 + \sum_n \frac{\boxed{k_n}}{s - q_n} = \frac{\prod_n (s - \boxed{p_n})}{\prod_n (s - q_n)}$$



These are the dominant poles!

S. Grivet-Talocia, “Package Macromodeling via Time-Domain Vector Fitting”, *IEEE Microwave Wireless Comp. Lett.*, Nov. 2003

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Time-Domain Vector Fitting

Step 2. Compute the residues

2a. Low-pass filter input signals with new poles

$$\boxed{\tilde{\mathbf{x}}_n(t) = \int_0^t e^{p_n (t-\tau)} \mathbf{x}(\tau)\, d\tau}$$

2b. Solve a linear least squares system for $\mathbf{R}_n$ and $\mathbf{H}_\infty$

$$\boxed{\mathbf{y}(t) = \mathbf{H}_\infty\, \mathbf{x}(t) + \sum_n \mathbf{R}_n\, \tilde{\mathbf{x}}_n(t)}$$

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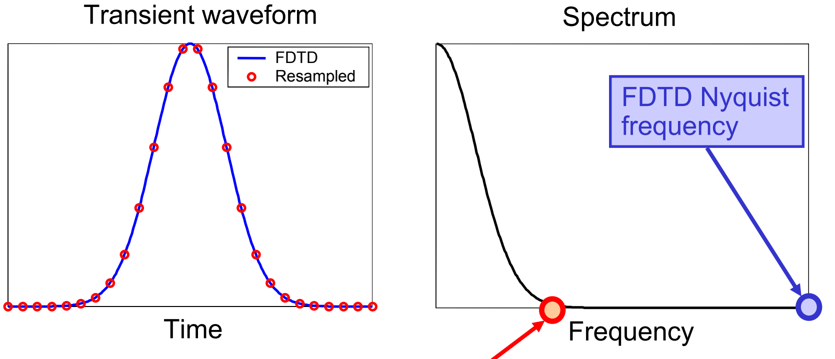
Subsampling



Nyquist frequency of resampled waveform = Effective bandwidth

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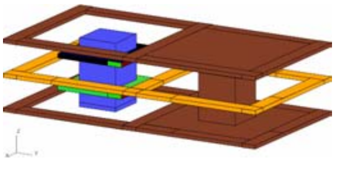
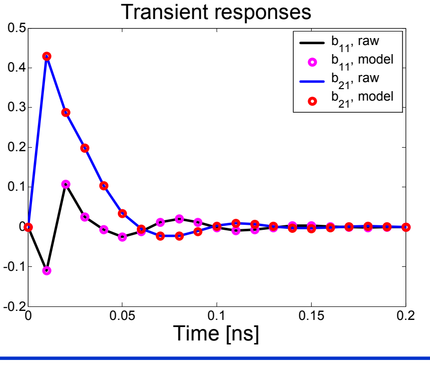
Example 1: single via



Raw data:
Triangle Impulse Responses obtained by a transient PEEC solver (by Dr. Ruehli, IBM)



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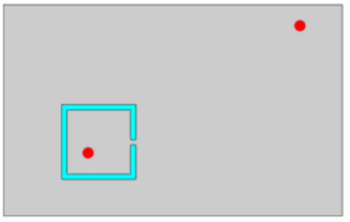
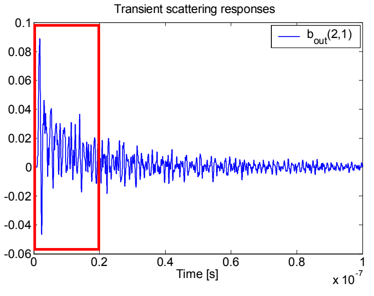
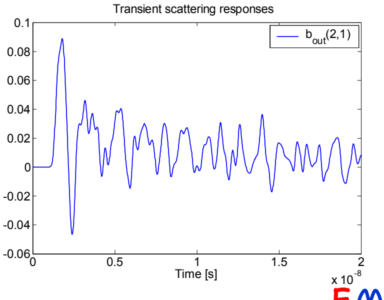
Example 2: segmented power bus



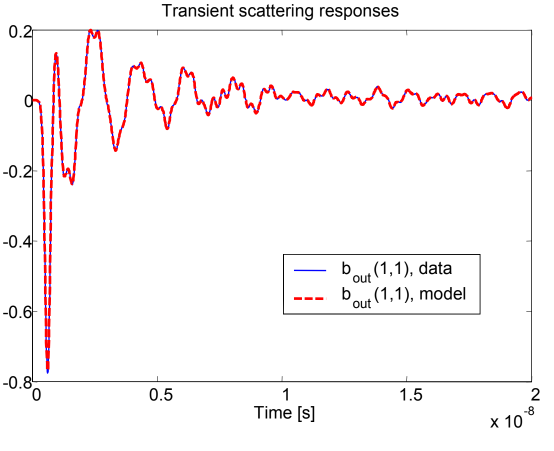
- 2-port structure
- Time-Domain solution
- CST Microwave Studio
- Bandwidth: 3 GHz
- 50 Ω port terminations





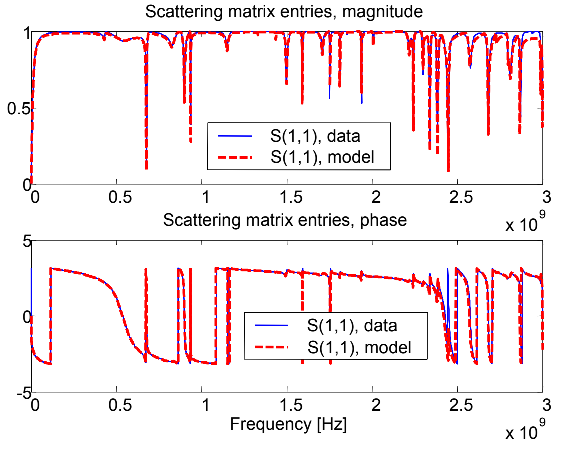
Example 2: segmented power bus

80-poles model (Time-Domain Vector Fitting)



Example 2: segmented power bus

Comparison vs. frequency-domain scattering data



Example 2: segmented power bus

Full-wave simulation time (CST) to compute…

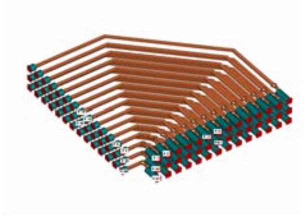
… frequency scattering data: 60 hours

(wait until transients are finished for reliable FFT)

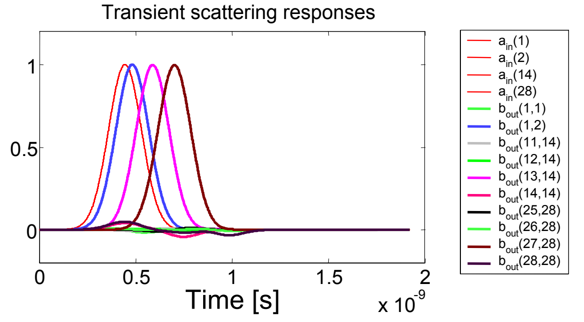
… macromodel: 6 hours

(can use truncated waveforms for TD-VF)

Example 3: 42-pin connector



3x14 pins, 84 ports
Characterized via FIT
(CST Microwave Studio 4)
(Courtesy: Erni - AdMOS)



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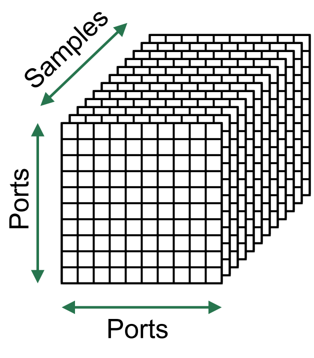
Handling many ports

Frequency-Domain Vector Fitting

$$\left(1 + \sum_n \frac{k_n}{s - q_n}\right)\mathbf{H}(s) = a + \sum_n \frac{b_n}{s - q_n}$$

Time-Domain Vector Fitting

$$\mathbf{y}(t) + \sum_n k_n\,\mathbf{y}_n(t) = a\,\mathbf{x}(t) + \sum_n b_n\,\mathbf{x}_n(t)$$



Processing **all** responses may lead to a **large** system!

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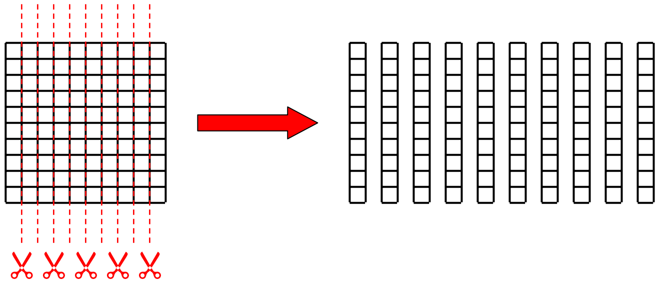
Handling many ports

1. Split port responses into subsets

Transfer matrix $\mathbf{H}(s)$ | Subsets $\{\mathbf{h}_k(s)\}$



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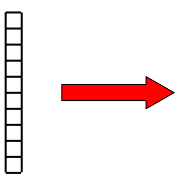
Handling many ports

2. Macromodel each subset via FD-VF or TD-VF



$$\mathbf{h}_k(s) \approx \mathbf{h}_{k,\infty} + \sum_n \frac{\mathbf{r}_{k,n}}{s - p_{k,n}}$$

Partial state-space representation

$$\begin{cases} \dot{\mathbf{w}}_k = \mathbf{A}_k \mathbf{w}_k + \mathbf{B}_k \mathbf{x}_k \\ \mathbf{y}_k = \mathbf{C}_k \mathbf{w}_k + \mathbf{D}_k \mathbf{x}_k \end{cases}$$

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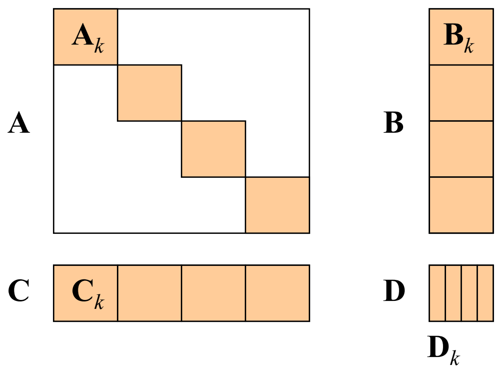
Handling many ports

4. Assemble all partial models into a global model

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{B}\;\mathbf{x} \\ \mathbf{y} = \mathbf{C}\,\mathbf{w} + \mathbf{D}\;\mathbf{x} \end{cases}$$

All matrices can be constructed as sparse!



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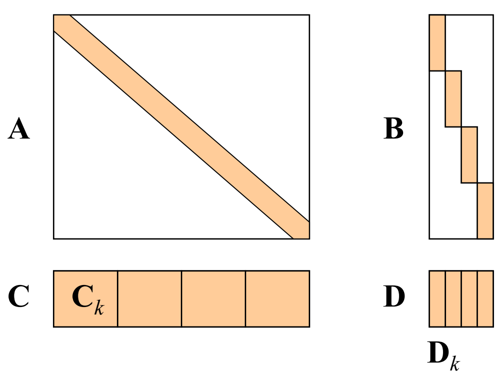
Handling many ports

4. Assemble all partial models into a global model

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{B}\;\mathbf{x} \\ \mathbf{y} = \mathbf{C}\,\mathbf{w} + \mathbf{D}\;\mathbf{x} \end{cases}$$

All matrices can be constructed as sparse!



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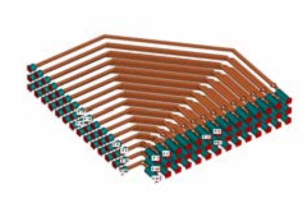
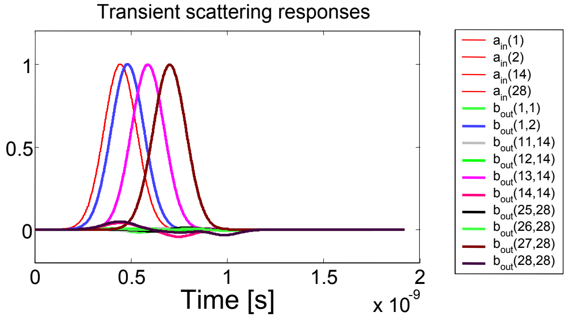
Example 3: 42-pin connector

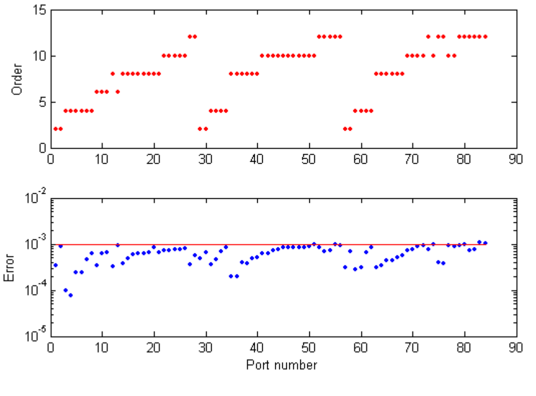


3x14 pins, 84 ports
Characterized via FIT
(CST Microwave Studio 4)
(Courtesy: Erni - AdMOS)

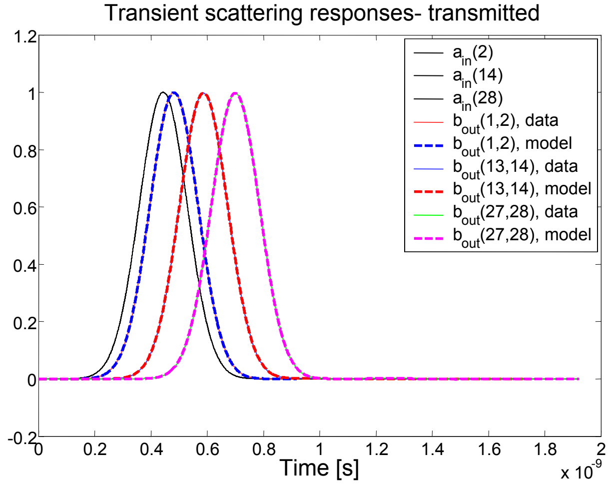


Example 3: model order selection

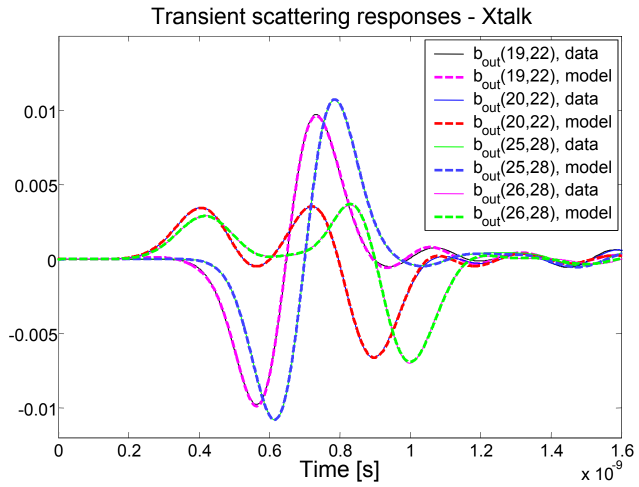
Automatic (iterative) order selection on each of the 84 subsets of port responses (reduced model complexity)



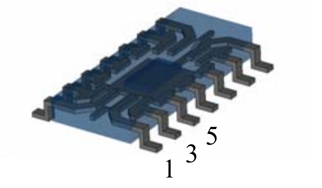
Example 3: macromodel accuracy



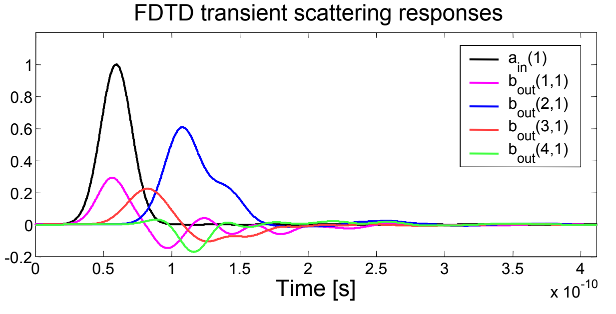
Example 3: macromodel accuracy



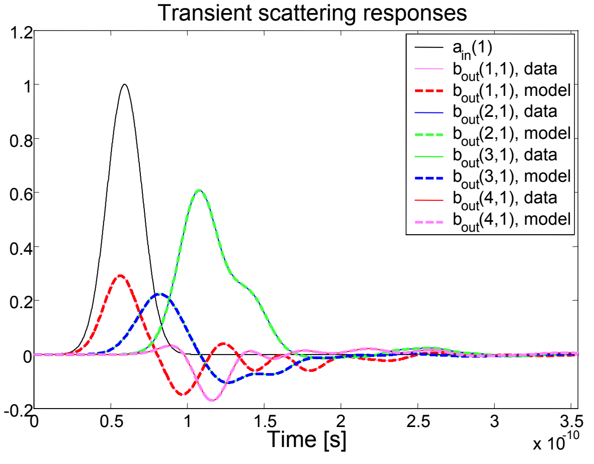
Example 4: 14-pin package



14-pin SOIC package
Simplified CAD for FDTD
Bandwidth: 40 GHz
50 Ω port terminations

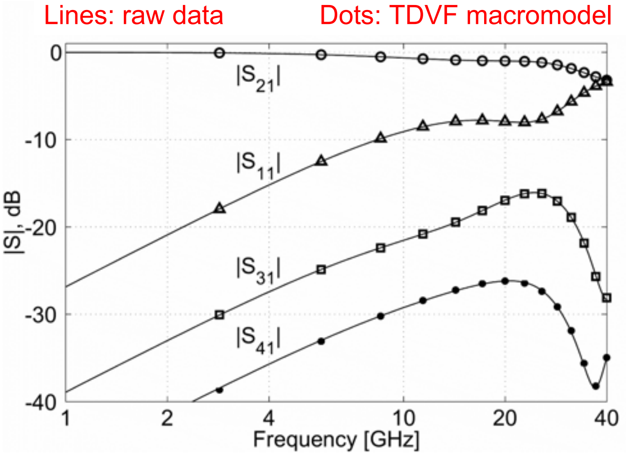


Example 4: macromodel responses



No visible difference between data and model

Example 4: macromodel responses



No visible difference between data and model

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Example 4: macromodel accuracy

Maximum deviation between model and data for all 28x28 responses

Largest: 0.00074

TD-VF produces highly accurate macromodels

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Macromodel properties

☺ **Accuracy**

Good initial data $\Rightarrow$ small approximation errors

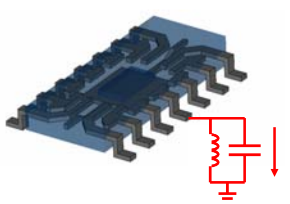
☺ **Stability**

All poles with negative real part

☹ **Passivity**

The macromodel may not be passive

Example 4: change terminations

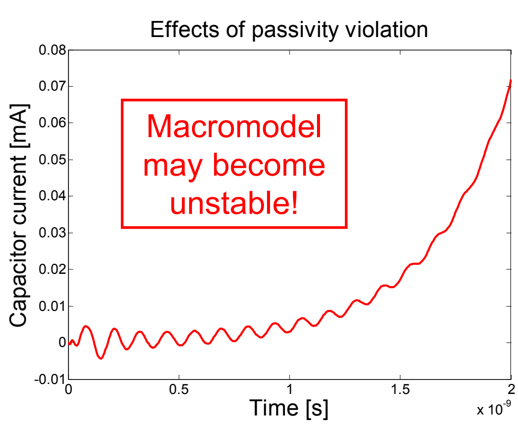


Port terminations:

$R = 50\ \text{m}\Omega \div 50\ \Omega$

$L = 1\ \text{nH}$

$C = 1\ \text{pF}$

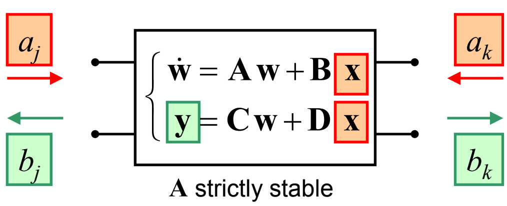


Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

Passivity conditions

Scattering representation



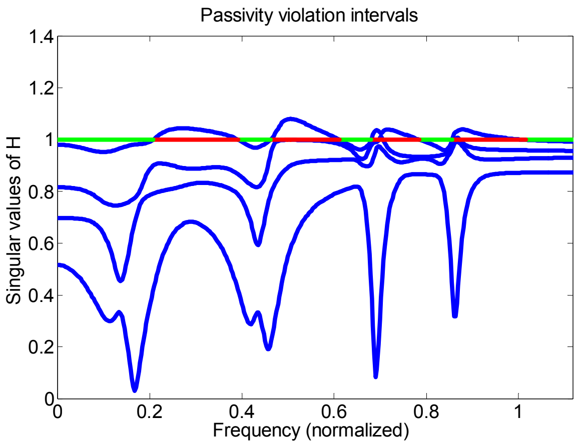
Scattering matrix: must be bounded real

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

$$\boxed{\{\text{singular values of } \mathbf{H}(j\omega)\} \leq \mathbf{1}, \quad \forall \omega}$$

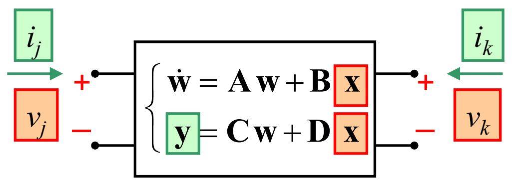
Passivity conditions

Scattering representation



Passivity conditions

Admittance representation



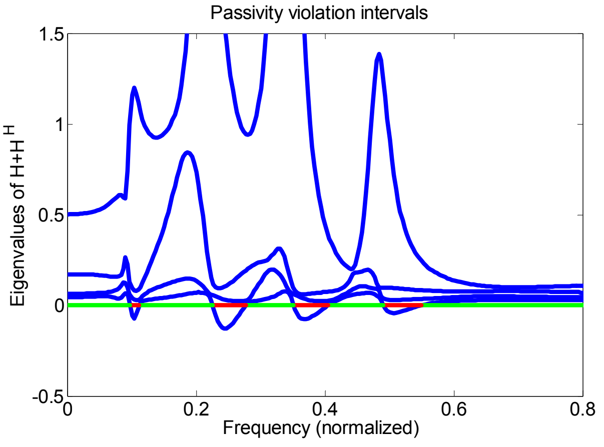
$\mathbf{A}$ strictly stable

Admittance matrix: must be positive real

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

$$\left\{ \text{eigenvalues of } \left(\mathbf{H}(j\omega) + \mathbf{H}^H(j\omega)\right) \right\} \geq \mathbf{0}, \quad \forall \omega$$

Passivity conditions
Admittance representation



Checking passivity
Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq \mathbf{1}, \quad \forall\omega$$

Several techniques can be used

Frequency sweep test: most straightforward

- Choose a set of frequency samples
- Compute $\mathbf{H}$ and its singular values, and check
- Time-consuming for large models
- May give wrong answers due to poor sampling

Checking passivity

Scattering representation

$$\boxed{\{\text{singular values of } \mathbf{H}(j\omega)\} \le \mathbf{1}, \quad \forall \omega}$$

Equivalent purely algebraic conditions:

- Linear Matrix Inequalities (**LMI**)
- Albegraic Riccati Equations (**ARE**)
- Eigenvalues of **Hamiltonian matrices**

S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan, "Linear Matrix Inequalities in System and Control Theory, SIAM, Philadelphia, 1994

Checking passivity

Scattering representation

$$\boxed{\{\text{singular values of } \mathbf{H}(j\omega)\} \le \mathbf{1}, \quad \forall \omega}$$

Linear Matrix Inequality (LMI)

$$\begin{pmatrix} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{C}^T\mathbf{C} & \mathbf{P}\mathbf{B} + \mathbf{C}^T\mathbf{D} \\ \mathbf{B}^T\mathbf{P} + \mathbf{D}^T\mathbf{C} & \mathbf{D}^T\mathbf{D} - \mathbf{I} \end{pmatrix} \le \mathbf{0} \qquad \mathbf{P} = \mathbf{P}^T, \mathbf{P} > 0$$

Real matrix $\mathbf{P}$ is the variable

Checking passivity

Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq \mathbf{1}, \quad \forall \omega$$

Algebraic Riccati Equation (ARE)

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{C}^T\mathbf{C} + \left(\mathbf{P}\mathbf{B} + \mathbf{C}^T\mathbf{D}\right)\left(\mathbf{I} - \mathbf{D}^T\mathbf{D}\right)^{-1}\left(\mathbf{P}\mathbf{B} + \mathbf{C}^T\mathbf{D}\right)^T = \mathbf{0}$$

$$\mathbf{P} = \mathbf{P}^T$$

Real matrix $\mathbf{P}$ is the variable

Checking passivity

Scattering representation

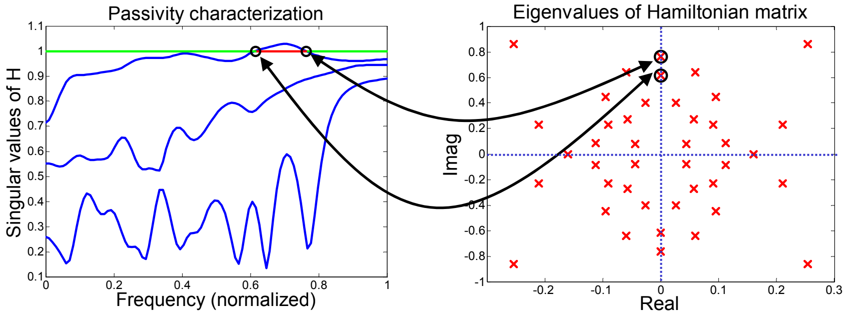
$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq \mathbf{1}, \quad \forall \omega$$

Eigenvalues of Hamiltonian matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} - \mathbf{B}\left(\mathbf{D}^T\mathbf{D} - \mathbf{I}\right)^{-1}\mathbf{D}^T\mathbf{C} & -\mathbf{B}\left(\mathbf{D}^T\mathbf{D} - \mathbf{I}\right)^{-1}\mathbf{B}^T \\ \mathbf{C}^T\left(\mathbf{D}\mathbf{D}^T - \mathbf{I}\right)^{-1}\mathbf{C} & -\mathbf{A}^T + \mathbf{C}^T\mathbf{D}\left(\mathbf{D}^T\mathbf{D} - \mathbf{I}\right)^{-1}\mathbf{B}^T \end{pmatrix}$$

Real matrix $\mathbf{M}$ must have no imaginary eigenvalues

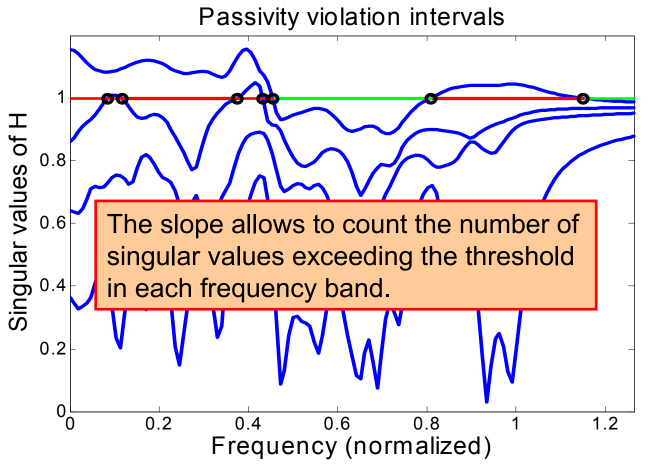
Checking passivity
Scattering representation



Theorem [Boyd, Balakrishnan, Kabamba, 1989]

$j\omega_0$ is an eigenvalue of $\mathbf{M} \Leftrightarrow \sigma = 1$ is a singular value of $\mathbf{H}(j\omega_0)$

Checking passivity
Scattering representation

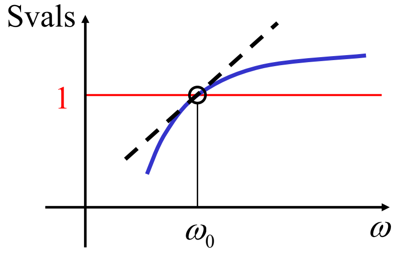


Checking passivity

Scattering representation

First-order perturbation of Hamiltonian eigenvalues

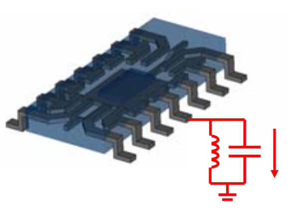
$$\boxed{Slope = \mathrm{Im}\left\{ \frac{\mathbf{w}^T \mathbf{v}}{\mathbf{w}^T \mathbf{M}' \, \mathbf{v}} \right\}}$$



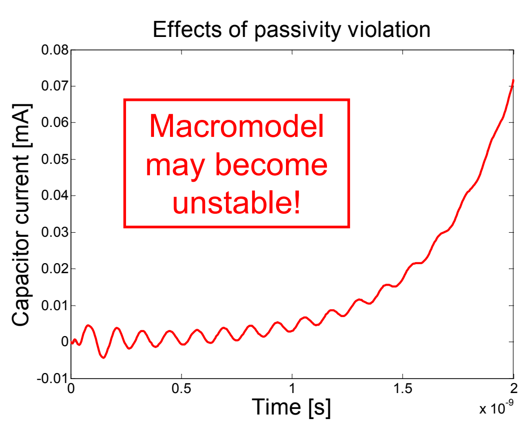
$\mathbf{w},\mathbf{v}$: Left and right eigenvectors of $\mathbf{M}$ associated to ω_0

$\mathbf{M}'$: Another Hamiltonian matrix (computed via $\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D}$)

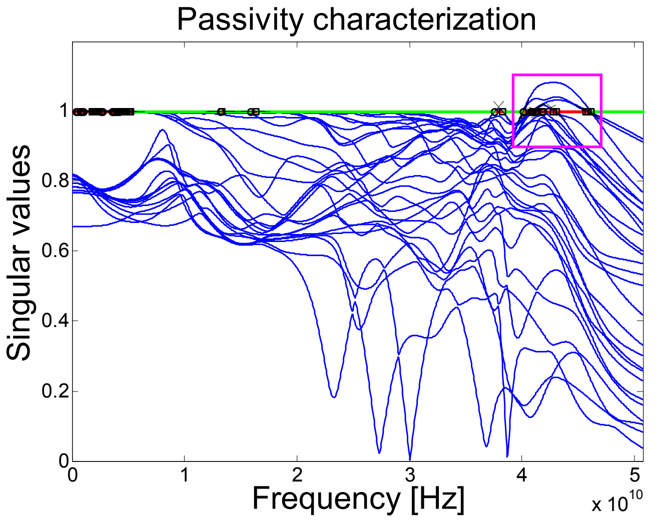
Example 4: change terminations



Port terminations:
R = 50 mΩ ÷ 50 Ω
L = 1 nH
C = 1 pF



Example 4: passivity characterization



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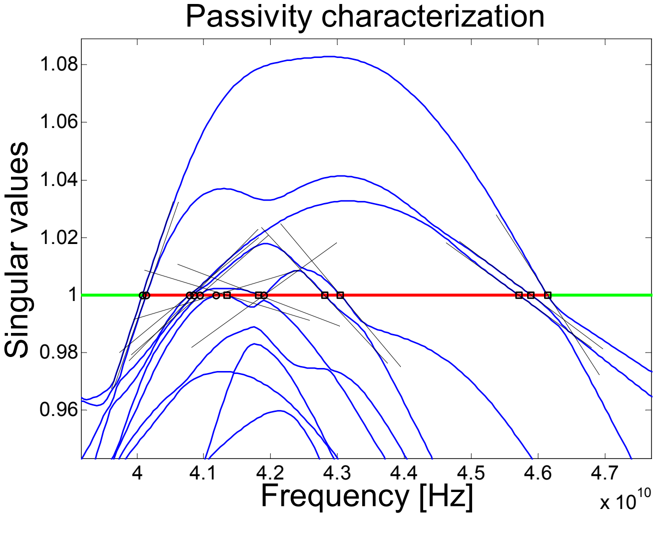
Example 4: passivity characterization



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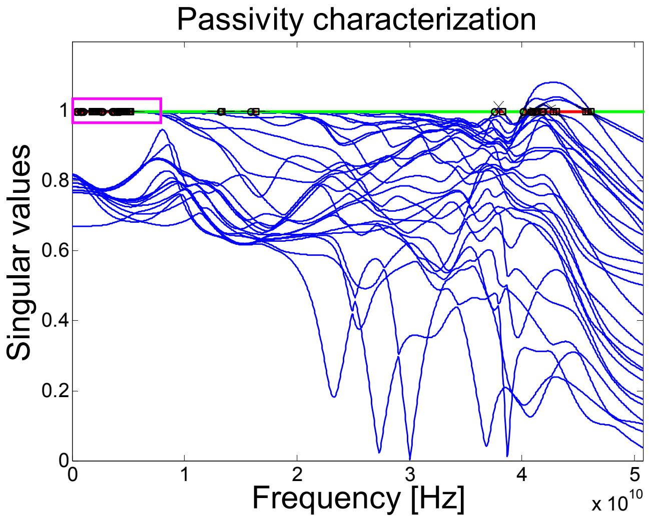
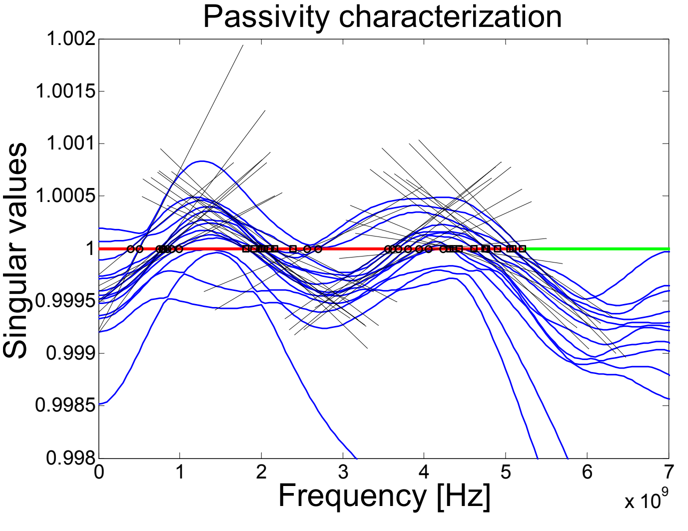
Example 4: passivity characterization



Example 4: passivity characterization



Passivity enforcement

- Generate a new passive macromodel
- Apply small correction to preserve accuracy
  - original dataset should be passive
  - original macromodel should be accurate
  - (usually) preserve poles

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{B}\;\mathbf{x} \\ \mathbf{y} = \mathbf{C}\,\mathbf{w} + \mathbf{D}\;\mathbf{x} \end{cases} \Rightarrow \begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{B}\;\mathbf{x} \\ \mathbf{y} = (\mathbf{C} + \mathbf{dC})\,\mathbf{w} + \mathbf{D}\;\mathbf{x} \end{cases}$$

Passivity enforcement

Several different approaches are possible. Examples are

Quadratic/convex optimization

[B.Gustavsen, A.Semlyen: IEEE Trans. Power Systems, vol.16, 2001]
[C.P.Coelho, J.Phillips, L.M.Silveira, IEEE Trans. CADICAS, vol.23, 2004]

Trace parameterization/Semi-Definite Programming

[H.Chen, J.Fang: Proc. EPEP, 2003]

Perturbation of residues

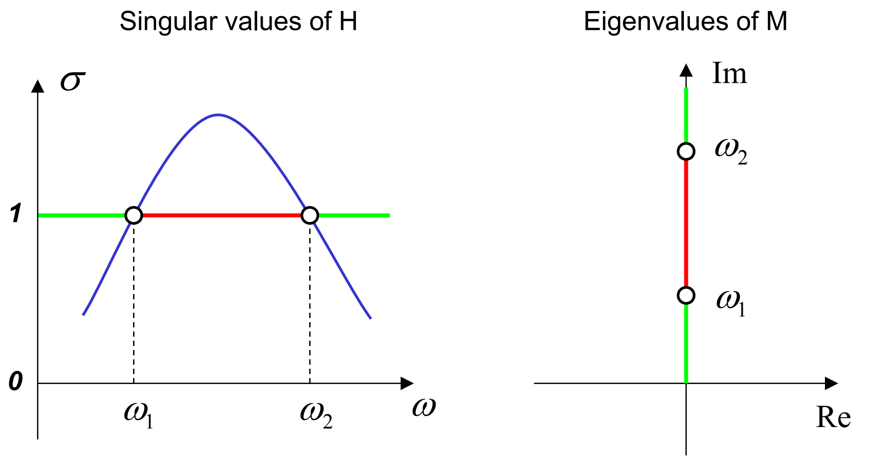
[D.Saraswat, R.Achar, M.Nakhla: Proc. EPEP, 2003]

Perturbation of Hamiltonian eigenvalues

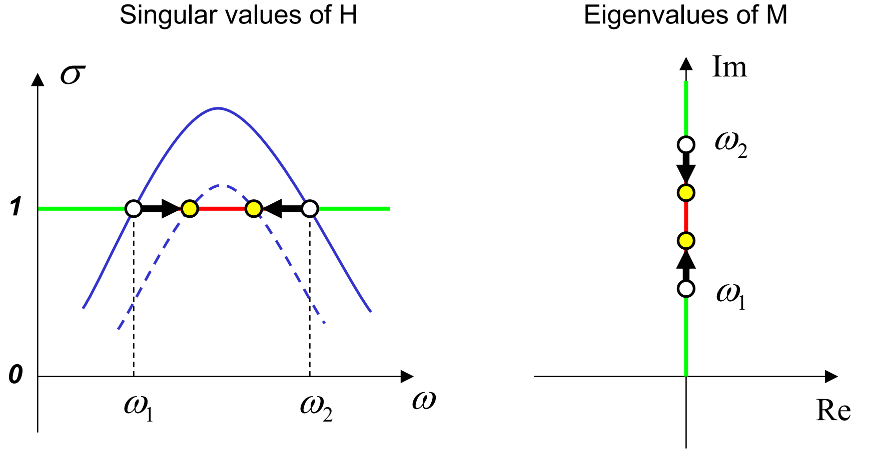
[S.Grivet-Talocia: Proc. SPI, 2003 and IEEE Trans. CAS (in press)]

Many others… Hot research topic!

Perturbation of Hamiltonian Eigs

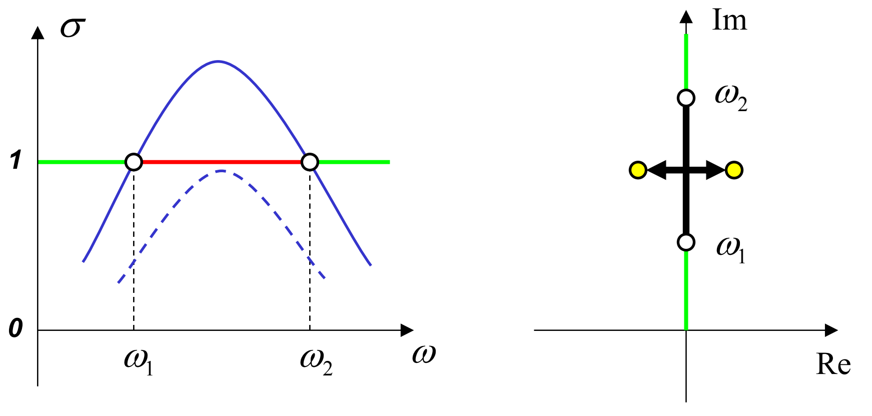


Perturbation of Hamiltonian Eigs



Perturbation of Hamiltonian Eigs

Singular values of H   Eigenvalues of M



EMC GROUP

Perturbation of Hamiltonian Eigs

First-order perturbation of eigenvalues (again)

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\ \mathbf{u} \\ \mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\ \mathbf{u} \end{cases}$$

Perturb state matrix $\mathbf{C}$

$$\tilde{\mathbf{C}} = \mathbf{C} + \mathbf{dC}$$

$\tilde{\mathbf{M}} \approx \mathbf{M} + \mathbf{dM}$ (first-order: $\mathbf{dM}$ is linear in $\mathbf{dC}$)

$$\mathbf{w}_m^T\,\mathbf{dM}\ \mathbf{v}_m \approx j(\tilde{\omega}_m - \omega_m)\,\mathbf{w}_m^T \mathbf{v}_m$$

Linear constraint on the correction matrix $\mathbf{dC}$

EMC GROUP

Perturbation of Hamiltonian Eigs

Iterate

$$\begin{cases} \boxed{\mathbf{Z}}\,\boxed{\mathbf{c}} = \boxed{\mathbf{r}} \\ \min\ \mathbf{c}^T \boxed{\mathbf{Q}}\,\mathbf{c} \end{cases}$$

Until passive

Linear coefficients

Entries in matrix $\mathbf{dC}$

Perturbations of eigenvalues

Related to controllability Gramian

Minimizes the perturbation on the original responses

Preserve accuracy of macromodel

Minimize this norm !

$$\sum_{i,j} \int_0^\infty \left(\tilde{h}_{i,j}(t) - h_{i,j}(t)\right)^2 \mathrm{d}t \qquad = \qquad \| \mathbf{dC}\ \mathbf{W}\ \mathbf{dC}^T \|_F^2$$

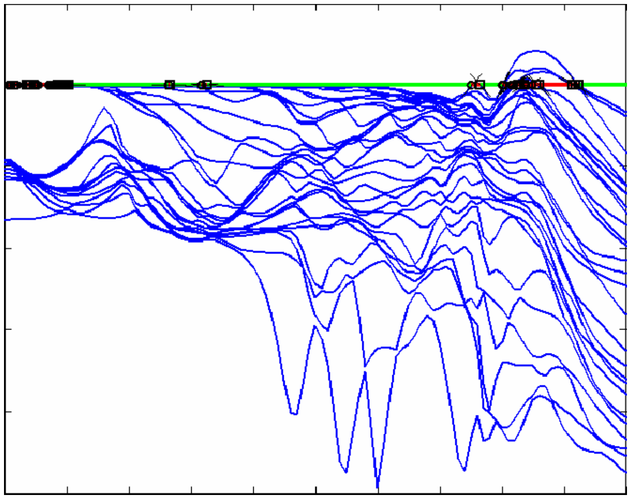
Induced perturbation in the impulse responses

Weighted norm of state matrix perturbation

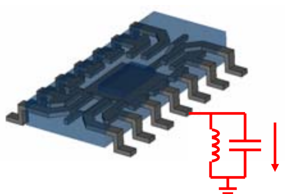
$\mathbf{W}$: controllability Gramian

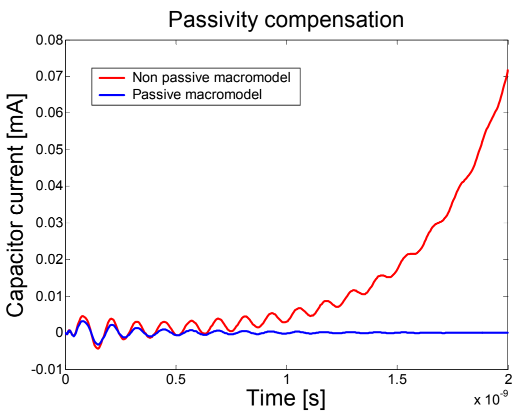
$$\mathbf{AW} + \mathbf{WA}^T = -\mathbf{BB}^T$$

Example 4: passivity compensation



Example 4 : passivity compensation





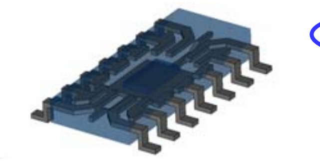
Port terminations:

$R = 50\ \mathrm{m}\Omega \div 50\ \Omega$

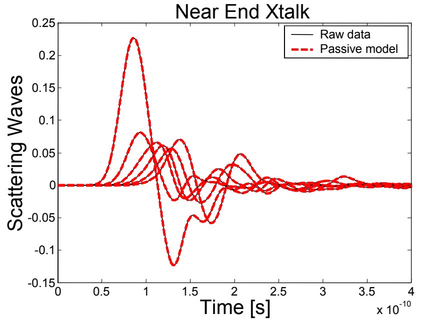
$L = 1$ nH

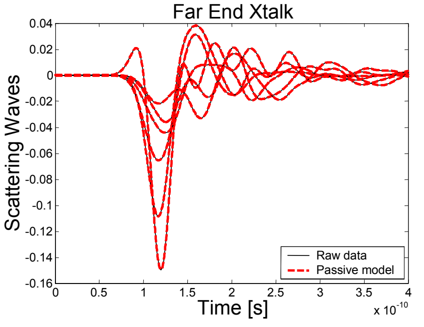
$C = 1$ pF

Example 4: passive macromodel

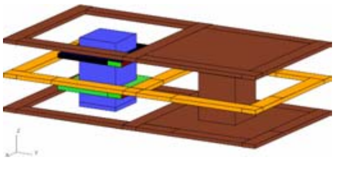


Raw data → Macromodel → Passive macromodel

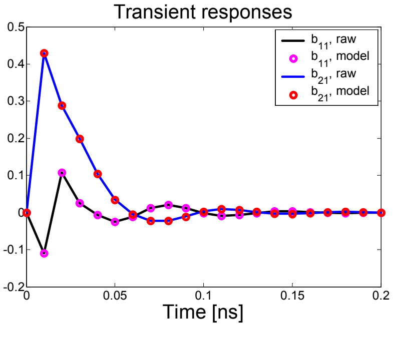


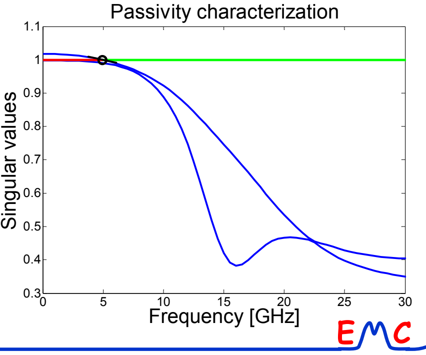


Example 1: single via, nonpassive

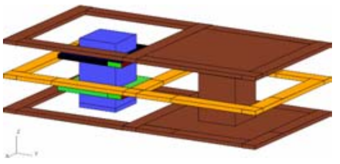


Raw data:
Triangle Impulse Responses obtained by a transient PEEC solver (by Dr. Ruehli, IBM)

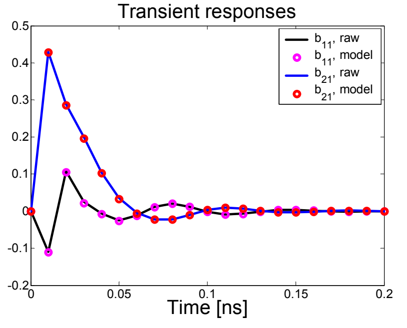


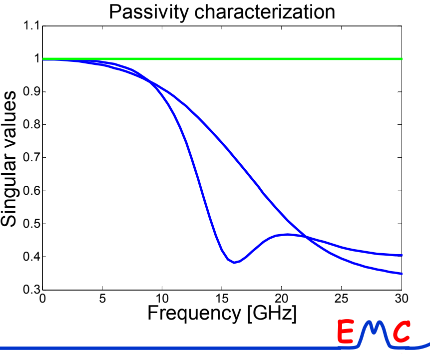


Example 1: single via, passive

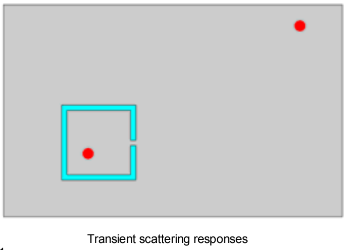


Raw data:
Triangle Impulse Responses obtained by a transient PEEC solver (by Dr. Ruehli, IBM)

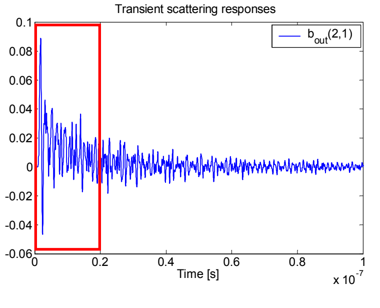


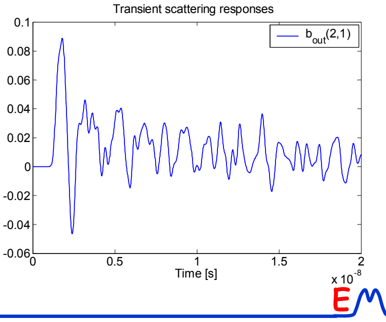


Example 2: segmented power bus



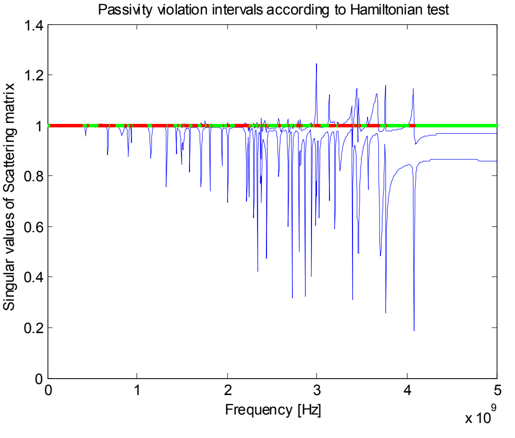
- 2-port structure
- Time-Domain solution
- CST Microwave Studio
- Bandwidth: 3 GHz
- 50 Ω port terminations





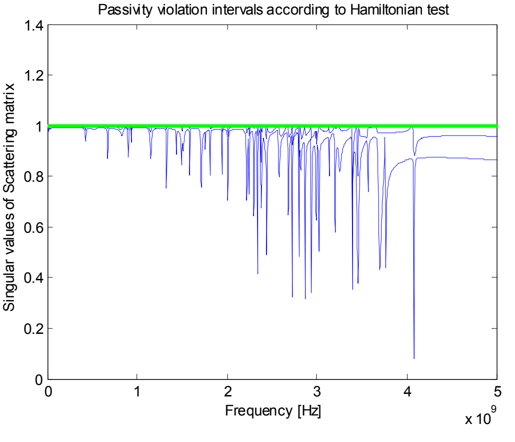
Example 2: segmented power bus

Passivity characterization



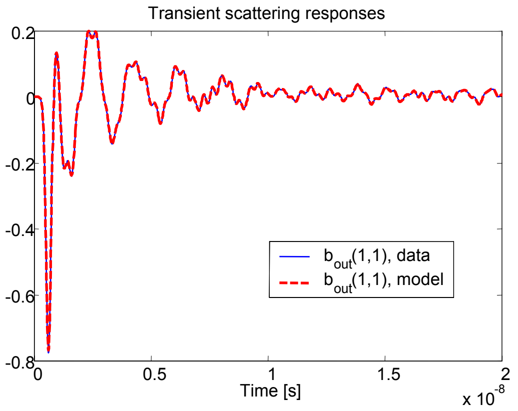
Example 2: segmented power bus

Passivity compensation



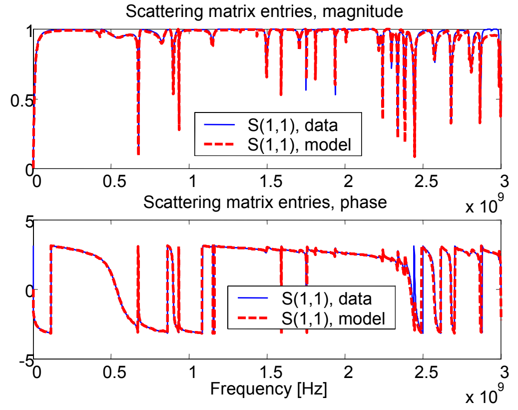
Example 2: segmented power bus

80-poles **passive** model



Example 2: segmented power bus

Comparison vs. frequency-domain scattering data



More examples…

41 poles, 2 ports

Compensation | Accuracy

110 poles, 5 ports

Compensation | Accuracy

308 poles, 11 ports

Compensation | Accuracy

Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
    - PRIMA
- Model Identification methods
    - Frequency-Domain Vector Fitting
    - Time-Domain Vector Fitting
    - Passivity characterization and enforcement
- SPICE synthesis

Macromodel implementation

Main approaches

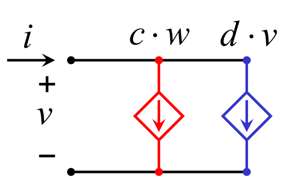
1. Synthesize an equivalent circuit in SPICE format
   - No access to SPICE kernel
   - Must use standard circuit elements
2. Direct SPICE implementation via recursive convolution
   - Laplace element, most efficient
3. Other languages for mixed-signal analyses
   - Verilog-AMS, VHDL-AMS, …
   - Equation-based

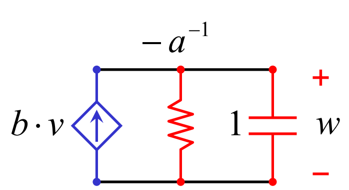
SPICE synthesis

Admittance representation

One-port, one-pole

$$\begin{cases} \dot{w} = a\,w + b\,v \\ i = c\,w + d\,v \end{cases}$$





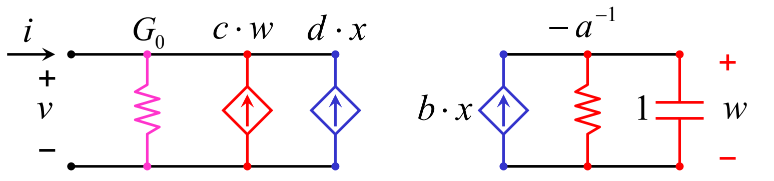
SPICE synthesis

Scattering representation

One-port, one-pole

$$\begin{cases} \dot{w} = a\,w + b\,x \\ y = c\,w + d\,x \end{cases}$$

$$x = G_0 v + i, \; y = G_0 v - i$$



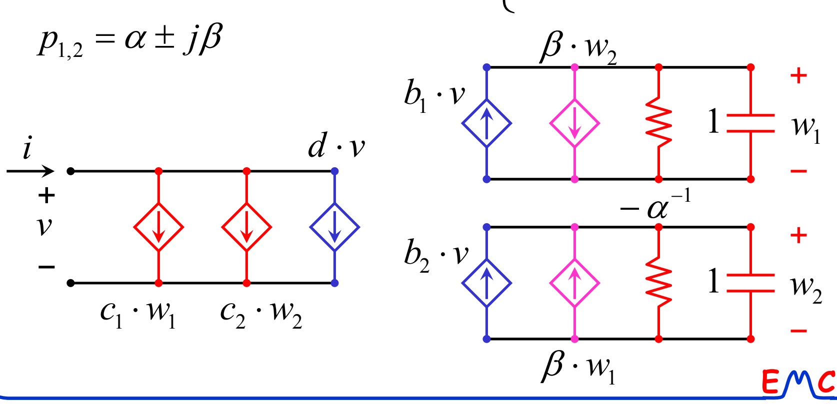
SPICE synthesis

Admittance representation

One-port, two-poles (complex)

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{b}\,v \\ i = \mathbf{c}\,\mathbf{w} + d\,v \end{cases}$$

$$p_{1,2} = \alpha \pm j\beta$$

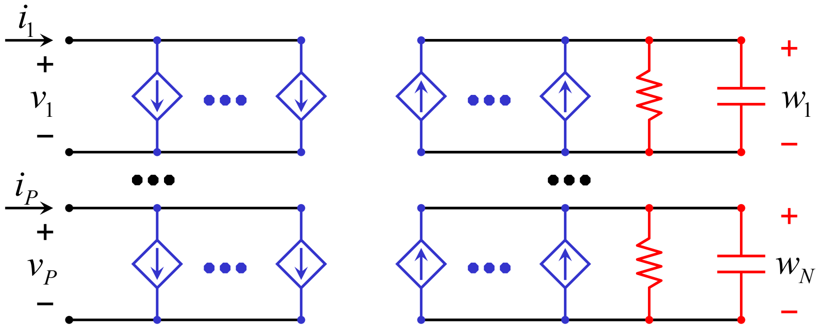


SPICE synthesis

Admittance representation

General state-space synthesis

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{B}\,\mathbf{v} \\ \mathbf{i} = \mathbf{C}\,\mathbf{w} + \mathbf{D}\,\mathbf{v} \end{cases}$$



Recursive convolutions

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} = \mathbf{D} + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

$$\mathbf{h}(t) = \mathbf{D}\delta(t) + \sum_n \mathbf{R}_n e^{p_n t} u(t)$$

$$\mathbf{y}(t) = \mathbf{D}\mathbf{x}(t) + \sum_n \mathbf{R}_n \int_0^t e^{p_n (t-\tau)} \mathbf{x}(\tau)\,\mathrm{d}\tau$$

Recursive convolutions

Discrete time

$$t_k = t_{k-1} + \Delta t_k$$

$$\begin{aligned}
\tilde{\mathbf{y}}(t_k) &= \int_0^{t_k} e^{p(t_k-\tau)}\mathbf{x}(\tau)\mathrm{d}\tau \\
&= \int_0^{t_{k-1}} e^{p(t_k-\tau)}\mathbf{x}(\tau)\mathrm{d}\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k-\tau)}\mathbf{x}(\tau)\mathrm{d}\tau \\
&= e^{p\Delta t_k}\int_0^{t_{k-1}} e^{p(t_{k-1}-\tau)}\mathbf{x}(\tau)\mathrm{d}\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k-\tau)}\mathbf{x}(\tau)\mathrm{d}\tau \\
&\approx e^{p\Delta t_k}\tilde{\mathbf{y}}(t_{k-1}) + \frac{1-e^{p\Delta t_k}}{p}\mathbf{x}(t_k)
\end{aligned}$$

Approximation!

The macromodeling dream…

Arbitrary characterization of the structure

- Equation-based or Black-Box
- Time or frequency, simulation or measurement

Generation of a broadband macromodel

- Any order, any number of ports
- Any prescribed accuracy
- Stable and passive by construction
- Efficient (reduced-order and low-complexity)
- Fully automatic