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Tutorial lecture on Characterization and Macromodeling of 3D Interconnects

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Characterization and Macromodeling of 3D Interconnects

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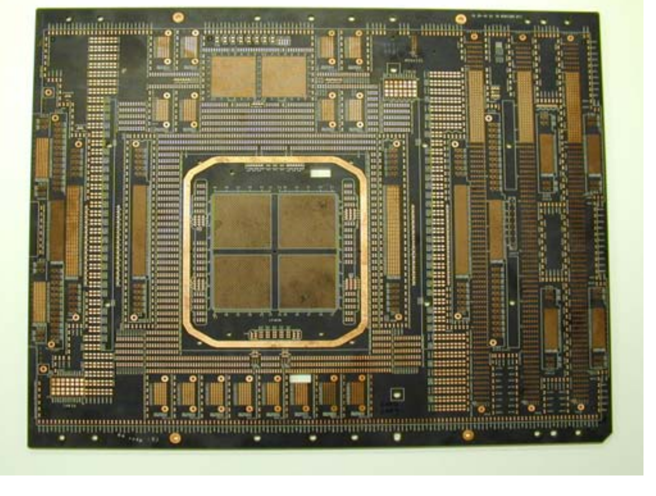
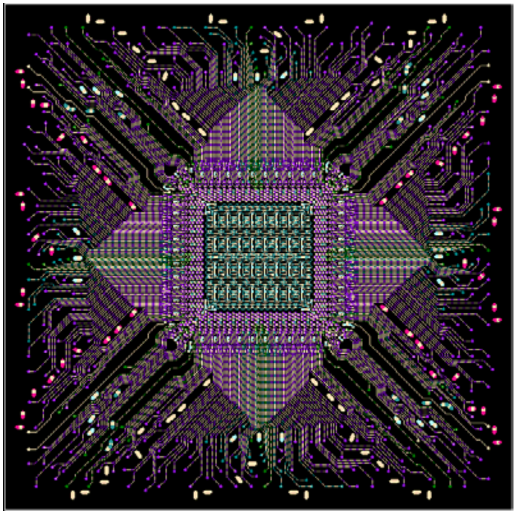
Introduction



Introduction



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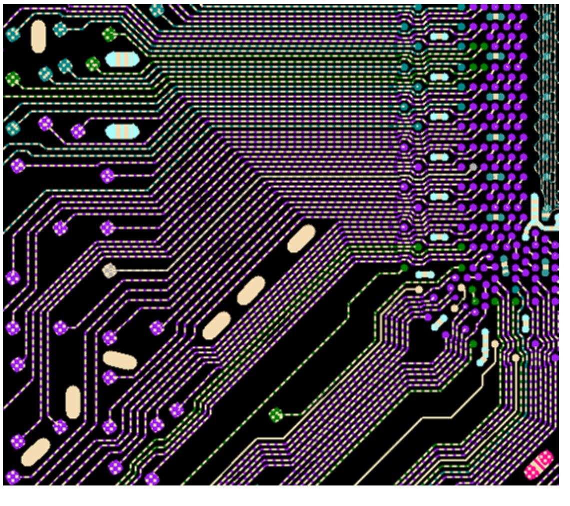
Introduction



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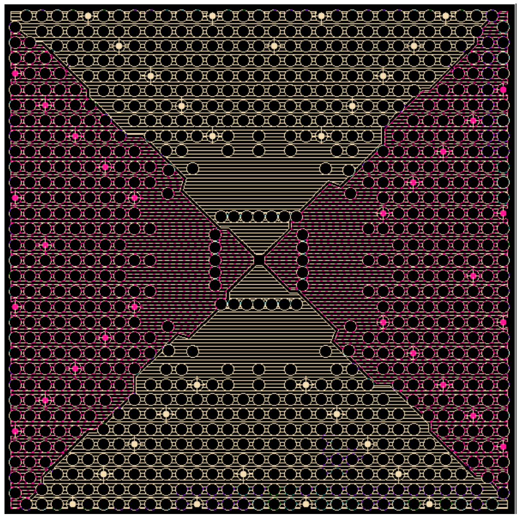
Introduction

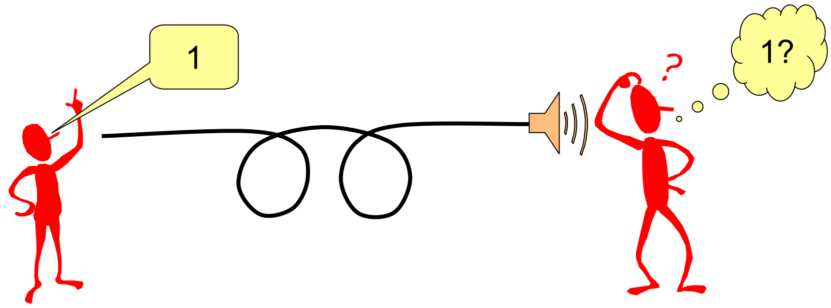


Introduction

High-speed Data transmission requires

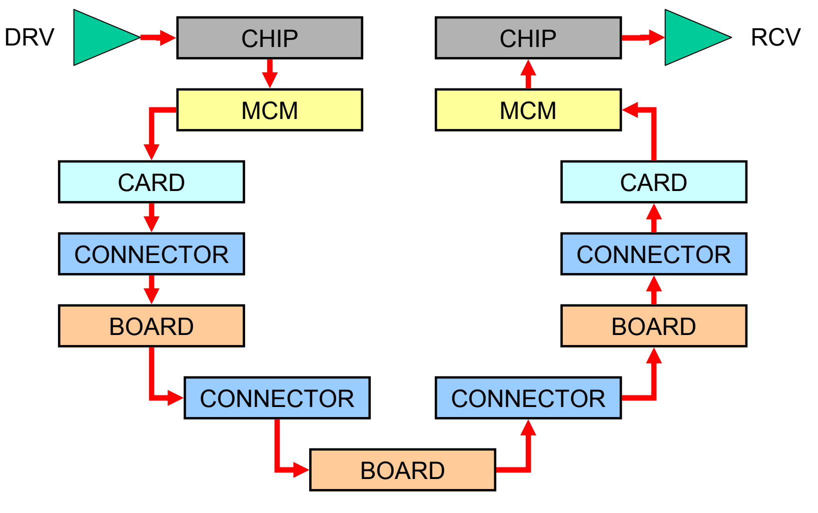
integrity of the signals

thru lines, bends, vias, connectors, …



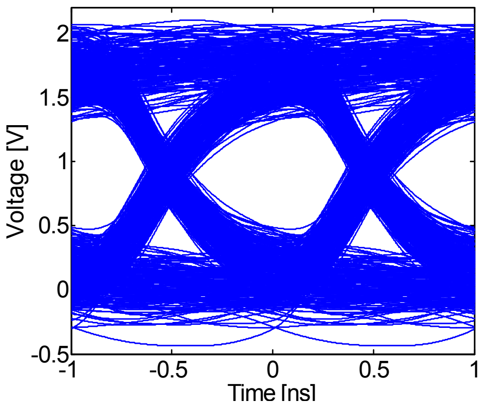
An example

An example



An example

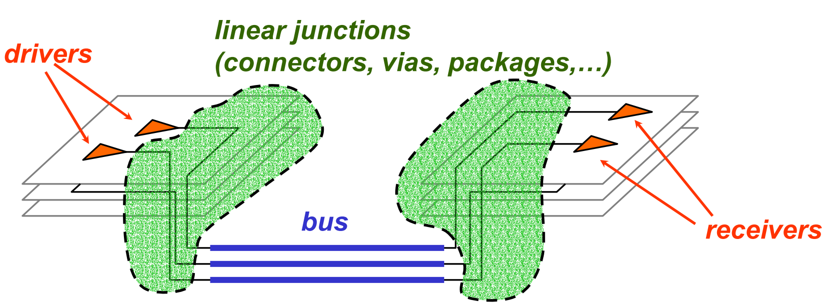
Transmission eye @ 1Gb/s (700 bits)



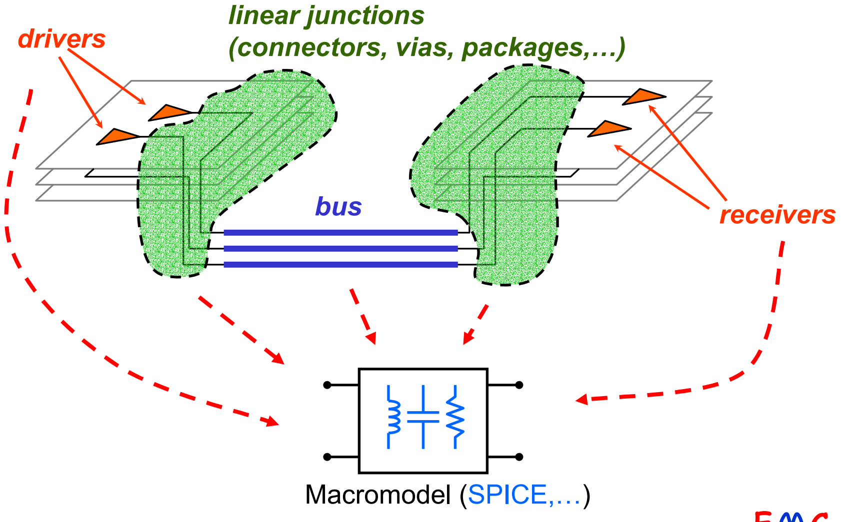
Introduction

Signal Integrity issues in high-speed digital systems

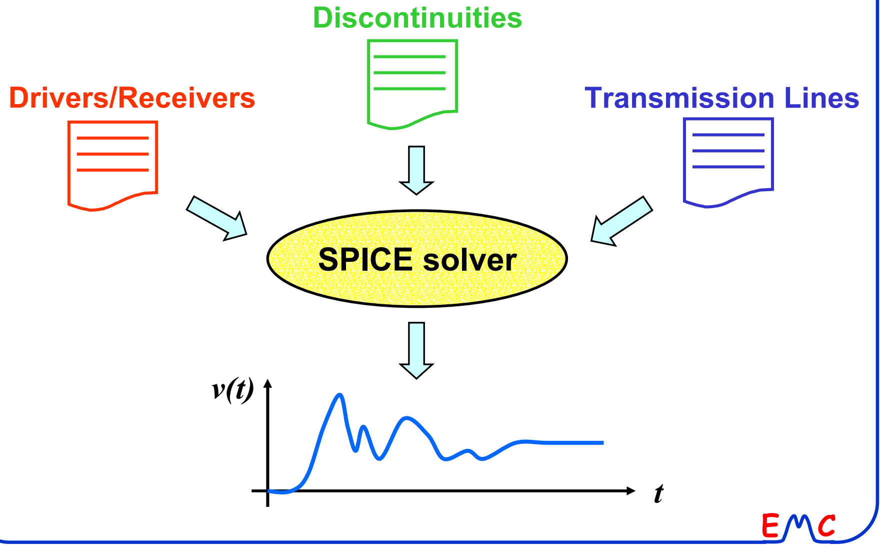
Crosstalk, couplings, reflections, losses, dispersion, attenuation, resonances, ground noise, nonlinear effects, radiation, EMI, …



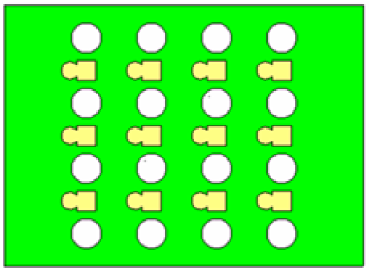
Introduction

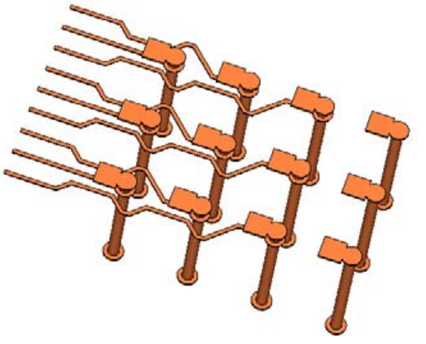


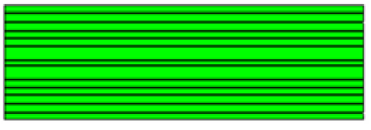
"Lego" approach

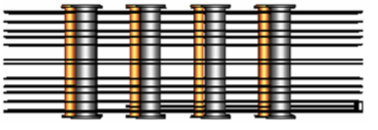


3D Interconnects

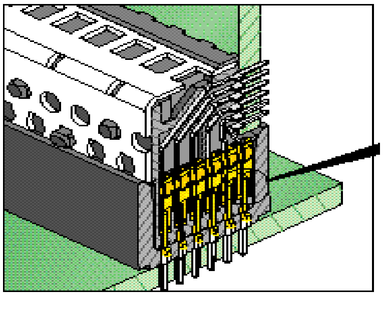


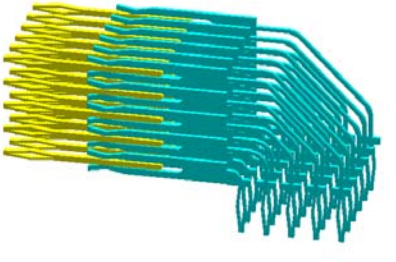




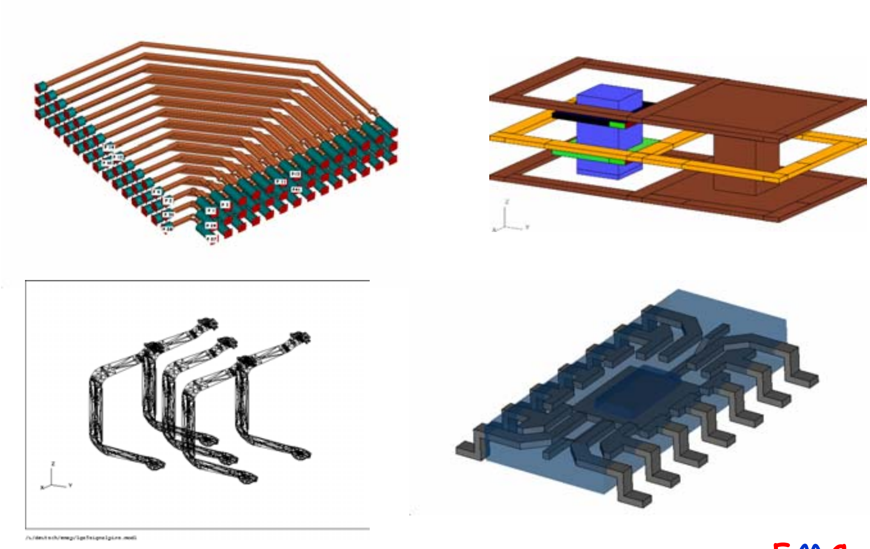


3D Interconnects





3D Interconnects



Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

Macromodeling approaches

Macromodeling of 3D interconnects for Signal Integrity

1. Capture physical effects leading to signal degradation
   - Must take into account 3D electromagnetic fields
     - Simulation or measurement
     - Many different characterizations are possible!
2. Use this information to build a macromodel
     - Many macromodeling approaches available!

Macromodeling approaches

Characterization via equations

Discretization of Maxwell full-wave equations

Model Order Reduction methods: build a simplified model from an existing (large) one

Characterization via port responses (Black Box)

Time or frequency domain

Simulated or measured

Reduced-Order Model Identification methods: build a model from samples of the port responses

Macromodeling approaches

Main goal of all (lumped) macromodeling methods:

produce a rational approximation

$$\mathbf{H}_q(s) = \mathbf{H}_\infty + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

Lumped circuits

- **have rational transfer functions**
- **are governed by Ordinary Differential Equations**

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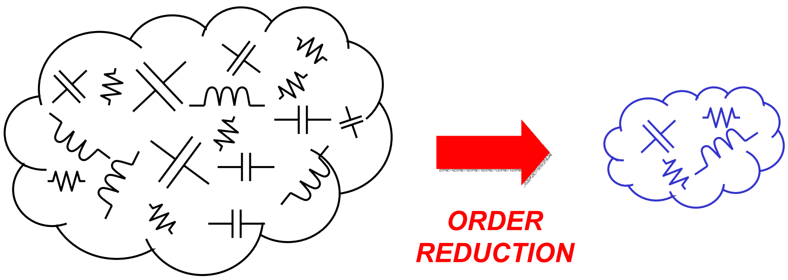
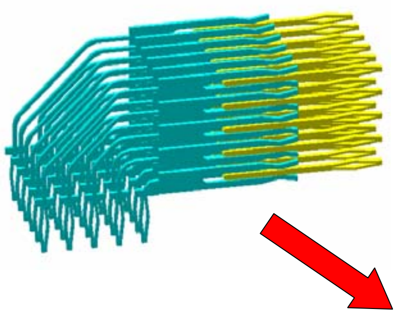
Moder Order Reduction

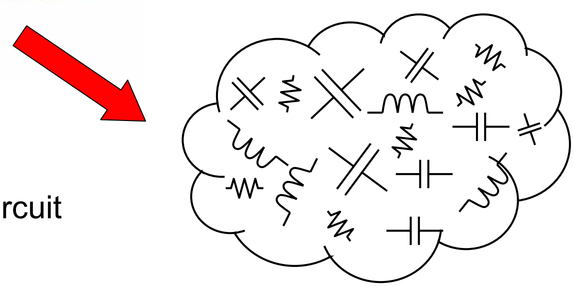


$$\mathbf{H}(s) \approx \mathbf{H}_q(s)$$

Possible scenarios

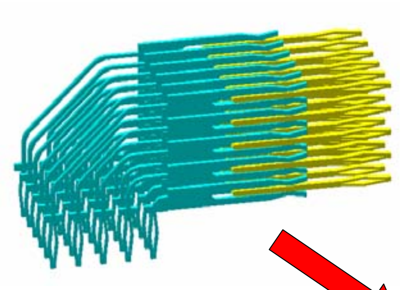


PEEC (Partial Element Equivalent Circuit) discretization



Large circuit

Possible scenarios



Spatial discretization of Maxwell equations (FDTD, FEM, MoM, …)

Large system

Set of ODEs (Ordinary Differential Equations)

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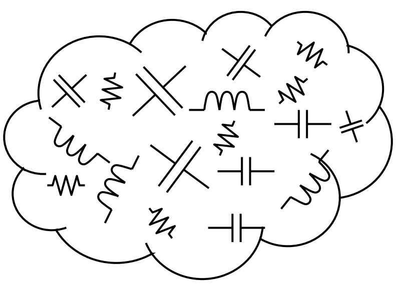
Possible scenarios



Set of ODEs (Ordinary Differential Equations)

MNA

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^{\mathrm{T}}\mathbf{x} \end{cases}$$

$$\mathbf{H}(s) = \mathbf{L}^{\mathrm{T}}(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B}$$

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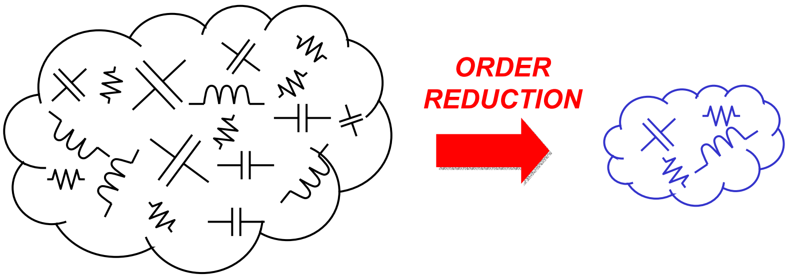
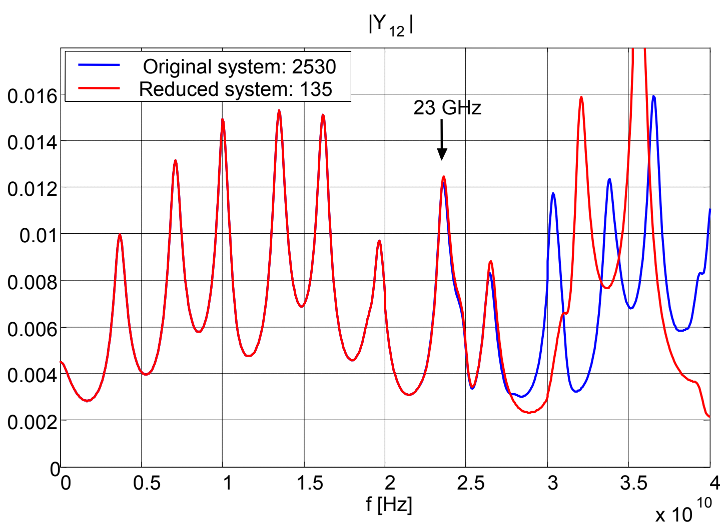
Approximation via moment matching



$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots + \mathbf{M}_N s^N + \cdots$$

$$\mathbf{H}_q(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots + \mathbf{M}_q s^q + \cdots$$

Moment matching: an example



Moments

$$\begin{cases} \mathbf{Gx} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{Bu} \\ \mathbf{y} = \mathbf{L}^{\mathrm{T}}\mathbf{x} \end{cases} \qquad \begin{cases} \mathbf{x} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{Ru} \\ \mathbf{y} = \mathbf{L}^{\mathrm{T}}\mathbf{x} \end{cases} \qquad \begin{array}{l} \mathbf{A} = -\mathbf{G}^{-1}\mathbf{C} \\ \mathbf{R} = \mathbf{G}^{-1}\mathbf{B} \end{array}$$

$$\mathbf{H}(s) = \mathbf{L}^{\mathrm{T}}(\mathbf{I} - s\mathbf{A})^{-1}\mathbf{Bu}$$

$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots$$

Moments $\quad \mathbf{L}^T\mathbf{R} \qquad \mathbf{L}^T\mathbf{A}\mathbf{R} \qquad \mathbf{L}^T\mathbf{A}^2\mathbf{R}$

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Moment matching techniques

Explicit $\quad \mathbf{M}_i = \mathbf{L}^T\mathbf{A}^i\mathbf{R} \quad \rightarrow \mathbf{H}_q(s)$

Asymptotic Waveform Evaluation (AWE)

Pade` Approximations

Complex Frequency Hopping (CFH)

- Good theoretical properties, convergence
- Bad numerical properties, intrinsic ill-conditioning due to
  - Moment generation
  - Moment matching

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Moment matching techniques

Implicit

Krylov subspace projection methods

- Same information stored in moments
- Much better numerical performance, robustness
- Several versions

  Arnoldi, PRIMA, Lanczos, …
- Possibility of preserving stability and passivity

  by construction!

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Krylov subspaces

$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots$$

Moments $\mathbf{L}^T\mathbf{R}$ $\quad \mathbf{L}^T\mathbf{A}\mathbf{R}$ $\quad \mathbf{L}^T\mathbf{A}^2\mathbf{R}$

$$\boxed{Kr(\mathbf{A},\mathbf{R},q) = \mathrm{span}\left\{\mathbf{R}, \mathbf{A}\mathbf{R}, \mathbf{A}^2\mathbf{R}, \ldots, \mathbf{A}^{q-1}\mathbf{R}\right\}}$$

$$\mathbf{V}_q = \text{basis of } Kr(\mathbf{A},\mathbf{R},q)$$

Constructed via iterative (stable) algorithms

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Arnoldi (basic) algorithm

$$\begin{cases} \mathbf{x} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{R}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^{\mathrm{T}}\mathbf{x} \end{cases}$$

$$\boxed{\mathbf{V}_q^T}\;\boxed{\mathbf{A}}\;\boxed{\mathbf{V}_q} = \boxed{\mathbf{A}_q}$$

$$\boxed{\mathbf{x}} \cong \boxed{\mathbf{V}_q}\;\boxed{\mathbf{x}_q}$$

$$\begin{cases} \mathbf{x}_q = \mathbf{A}_q\dot{\mathbf{x}}_q + \mathbf{R}_q\mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^T\mathbf{x}_q \end{cases}$$

PRIMA algorithm

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^{\mathrm{T}}\mathbf{x} \end{cases}$$

$$\boxed{\mathbf{V}_q^T}\;\boxed{\mathbf{C}}\;\boxed{\mathbf{V}_q} = \boxed{\mathbf{C}_q}$$

$$\boxed{\mathbf{V}_q^T}\;\boxed{\mathbf{G}}\;\boxed{\mathbf{V}_q} = \boxed{\mathbf{G}_q}$$

$$\boxed{\mathbf{x}} \cong \boxed{\mathbf{V}_q}\;\boxed{\mathbf{x}_q}$$

$$\begin{cases} \mathbf{G}_q\mathbf{x}_q + \mathbf{C}_q\dot{\mathbf{x}}_q = \mathbf{B}_q\mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^T\mathbf{x}_q \end{cases}$$

Passivity conditions (PRIMA)

$$\begin{cases} \mathbf{Gx} + \mathbf{C\dot{x}} = \mathbf{Bu} \\ \mathbf{y} = \mathbf{L}^{\mathrm{T}}\mathbf{x} \end{cases}$$

Can often be enforced by construction building the original system

$$\begin{cases} \mathbf{G}_q\mathbf{x}_q + \mathbf{C}_q\dot{\mathbf{x}}_q = \mathbf{B}_q\mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^T\mathbf{x}_q \end{cases}$$

$\mathbf{G} \geq 0$ $\quad$ $\mathbf{C} \geq 0$

$\mathbf{C}$ symmetric

$\mathbf{L} = \pm\mathbf{B}$

$\mathbf{V}_q$ must be full rank

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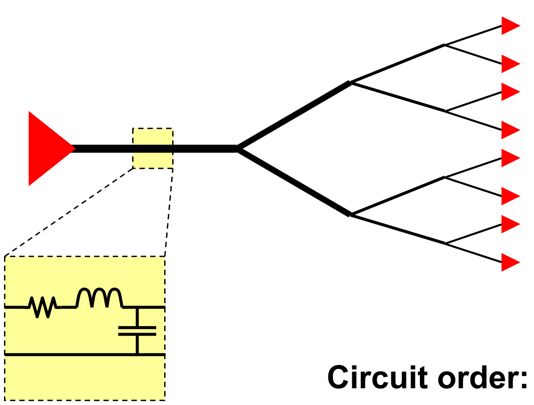
An example: RLC tree circuit



Circuit order: 2530

Ports: 9

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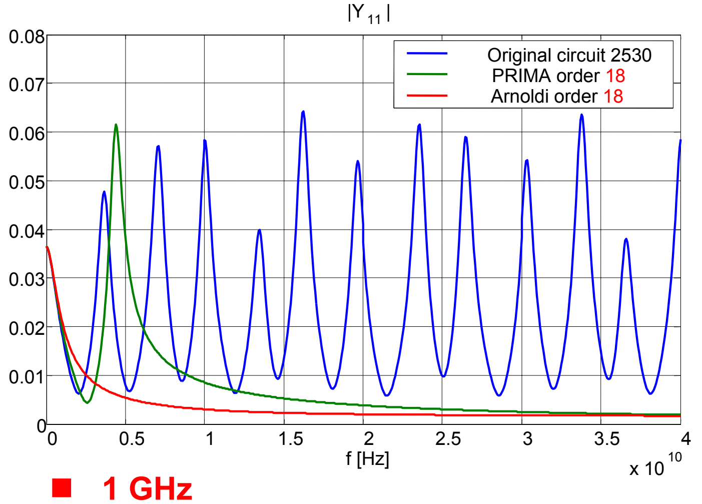
RLC tree circuit: order reduction



- **1 GHz**

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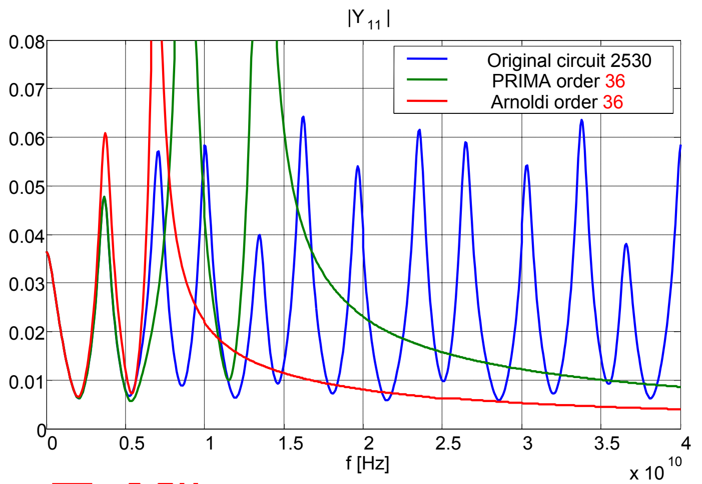
RLC tree circuit: order reduction



- **2 GHz**

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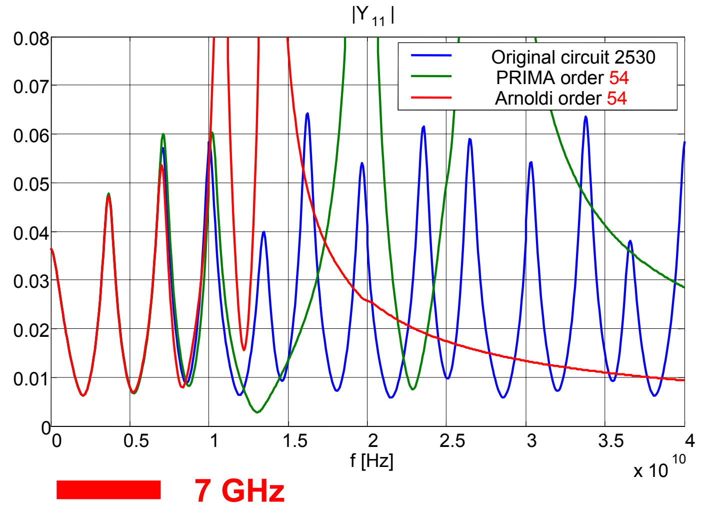
RLC tree circuit: order reduction



7 GHz

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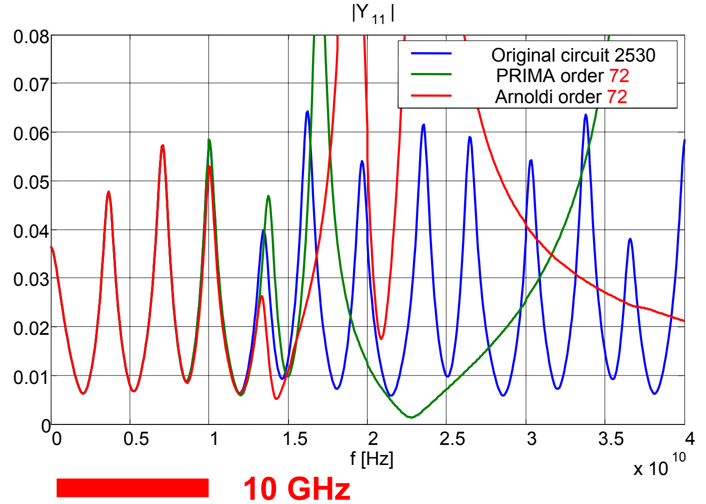
RLC tree circuit: order reduction



10 GHz

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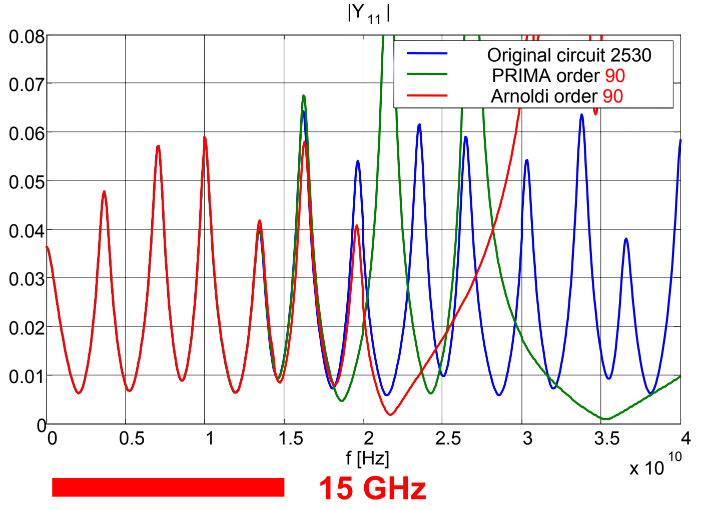
RLC tree circuit: order reduction



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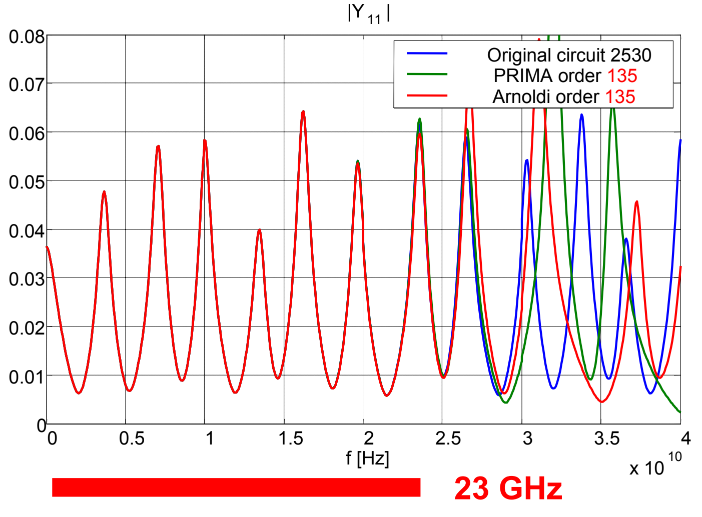
RLC tree circuit: order reduction



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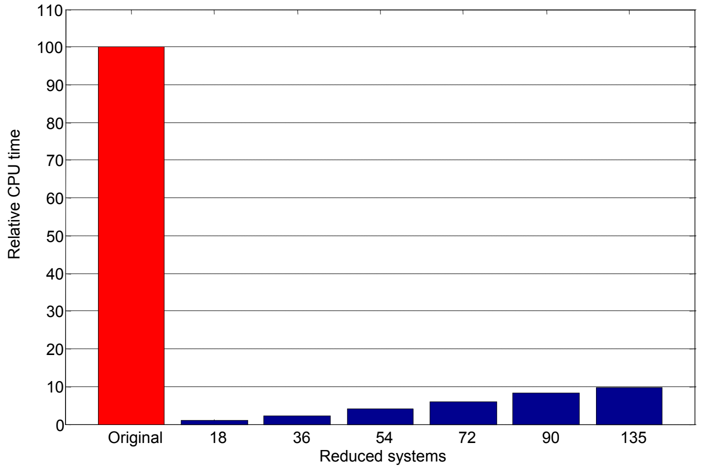
RLC tree circuit: efficiency



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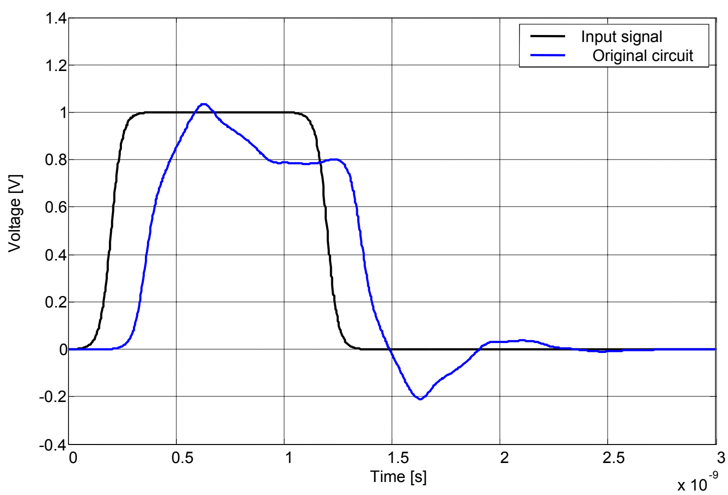
RLC tree circuit: transient analysis



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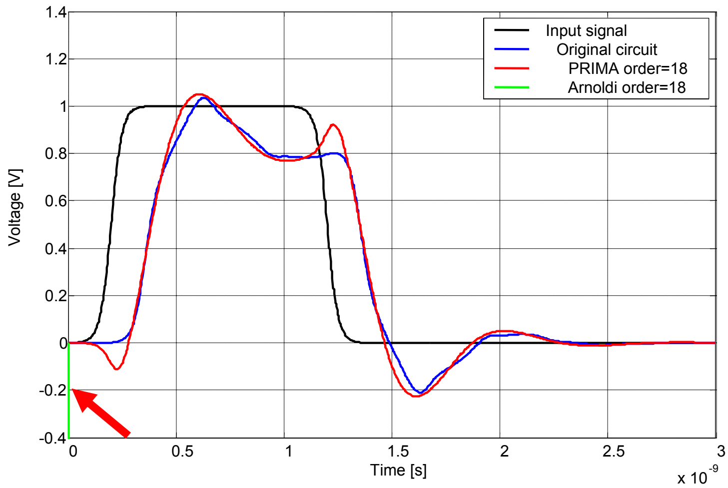
RLC tree circuit: transient analysis



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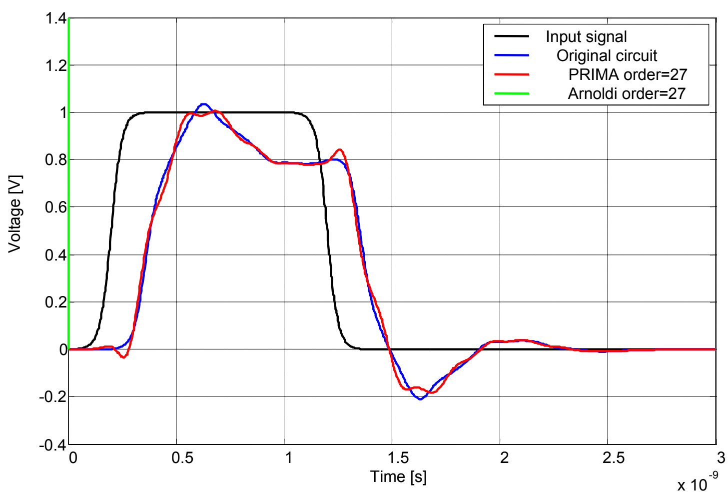
RLC tree circuit: transient analysis



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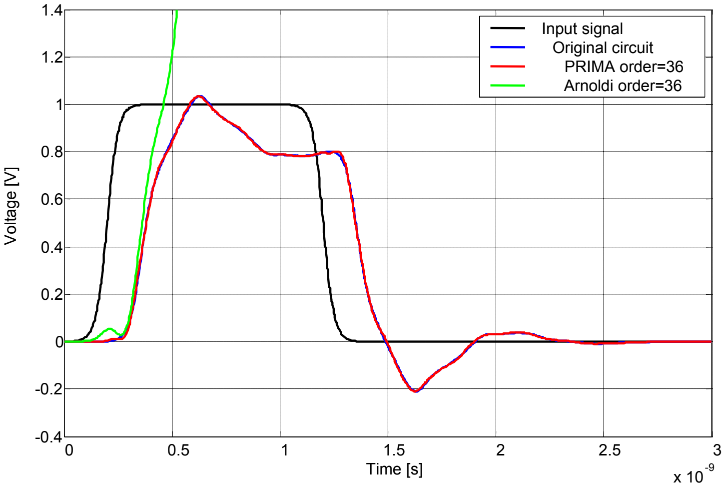
RLC tree circuit: transient analysis



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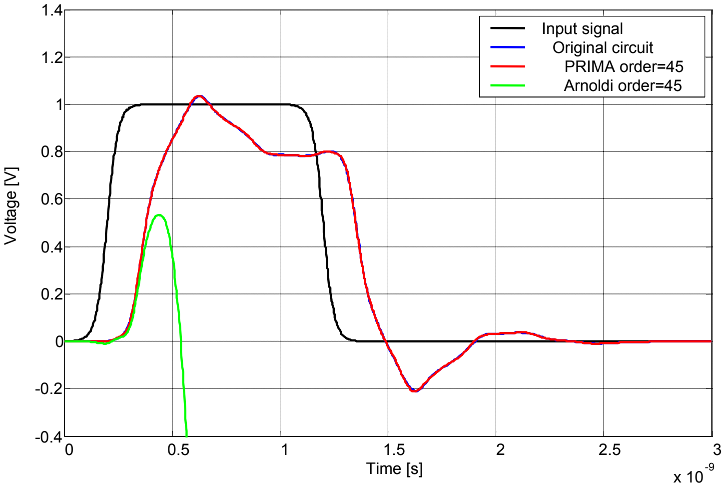
RLC tree circuit: transient analysis



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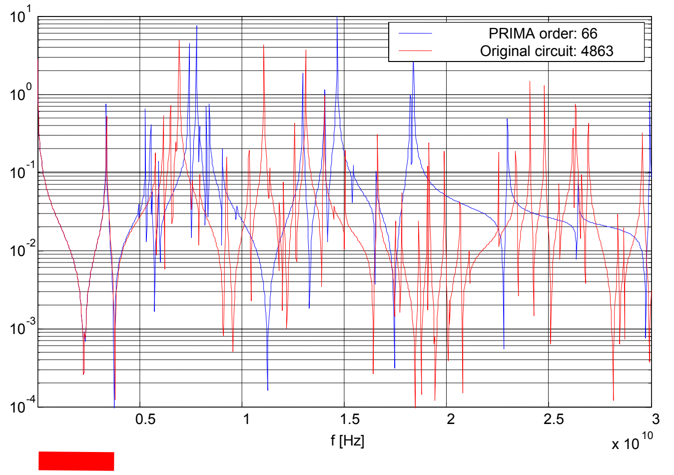
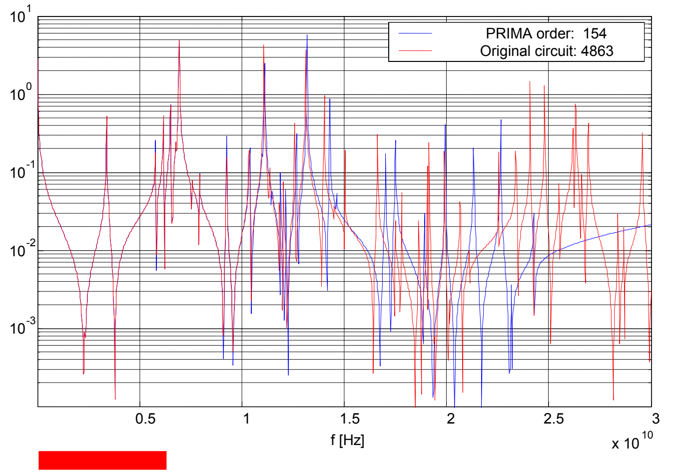
Example: MNA, 22 ports, order 4863

(www.win.tue.nl/niconet/NIC2/benchmodred.html)



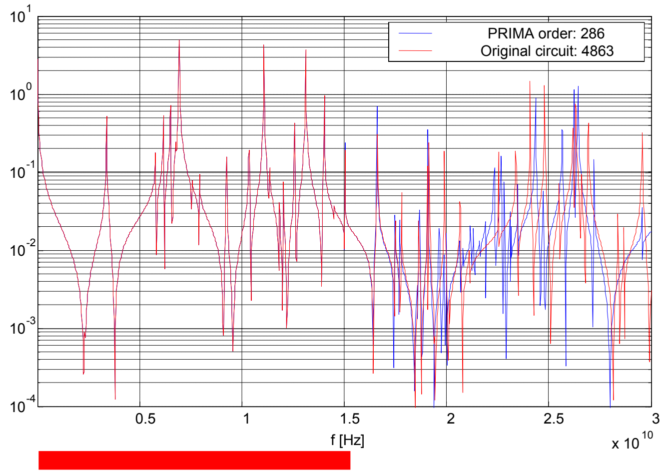
3 GHz

Example: MNA, 22 ports, order 4863



6 GHz

Example: MNA, 22 ports, order 4863



15 GHz

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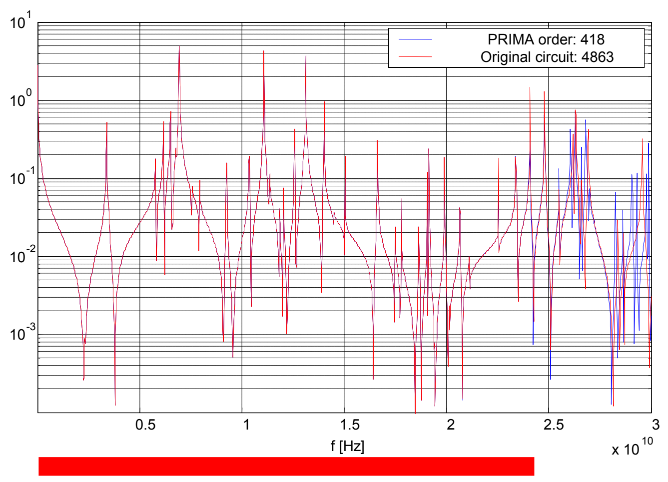
Example: MNA, 22 ports, order 4863



24 GHz

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Key references

M.Celik, L.Pileggi, A.Odabasioglu, IC Interconnects Analysis, Kluwer, 2002

…and references therein

R.Achar, M.S.Nakhla, Proceedings of the IEEE, Vol.89, 2001, 693-728

… and references therein

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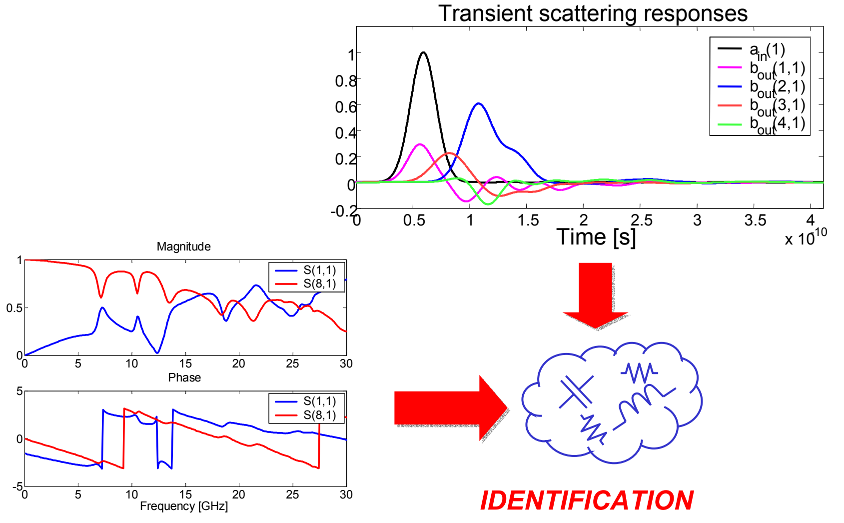
Model Identification



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Model identification

From samples to model: identification process

Reduced-order identification: approximation process

Several identification methods exist

Characterized by use of different:

- Input data
- Modeling criteria
- Model parameter estimation

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Identification methods

Block Complex Frequency Hopping (BCFH)

[R.Achar, M.S.Nakhla, IEEE Proceedings, Vol.89, 2001]

- Rational Padé approximation of network functions
- Convergence property in a neighborhood of the expansion point
- Hopping along frequency axis to cover the modeling bandwidth
- May lead to ill-conditioned numerical systems when used for identification from sampled responses

Identification methods

Global Rational Approximation

[M.Elzinga, K.L.Virga, J.L.Prince, IEEE Trans. MTT, 9/2000]

[W.Beyene, J.Schutt-Aine, IEEE Trans. CPMT, Vol.21, 3/1998]

[K.L.Choi, M.Swaminathan, IEEE Trans. CAS II, vol.47, 4/2000]

[J.Morsey, A.C.Cangellaris, Proc. EPEP, 2001]

[… many, many, many others…]

- A matrix of rational functions is fitted to the samples of a network function matrix (e.g. the Y matrix)

Identification methods

Pencil of Functions

[Y.Hua, T.Sarkar, IEEE Trans. AP, vol.37, 2/1989]

- Time-domain data
- Estimates model poles by fitting a sum of exponential functions to the samples of transient port responses
- Poles obtained as eigenvalues of a generalized eigenvalue problem
- Automatic order estimation

Identification methods

Subspace-based State-Space System Identification methods (4SID)

[M. Viberg, Automatica, 12/1995]

[T.McKelvey, H.Akcay, L.Ljung, IEEE Trans. AC, vol.41, 7/1996]

- Based on projections of data onto orthogonal subspaces, leading to direct state-space estimation
- Built-in automatic order estimation (based on SVD)
- Available in both time and frequency domain
- Equivalent to Pencil of Functions methods

Identification methods

Nevanlinna-Pick Interpolation

[C.P.Coelho, J.R.Phillips, L.M.Silveira, Proc. DATE 2002]

- Interpolation of samples of the scattering matrix with a (unitary bounded) matrix rational function
- Nice theoretical properties
- Very complex
- Leads to models with large dynamical order

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Identification methods

Vector Fitting

[B.Gustavsen, A. Semlyen, IEEE Trans. PD, vol.14, 7/1999]

- Performs data fitting with rational functions avoiding nonlinear optimization
- Iterative process converging to the dominant poles
- Available for both time and frequency domain

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Identification methods

Identification methods are not expected to work for every possible problem

Any method performs well for a certain class of identification problems

Vector Fitting is selected here as one of the most promising methods for a wide range of applications

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Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

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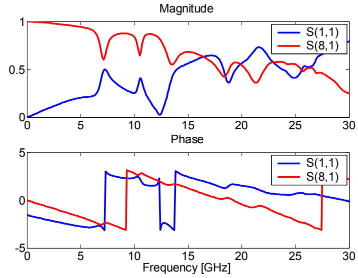
Frequency-domain macromodeling

Model identification from frequency-domain responses



IDENTIFICATION

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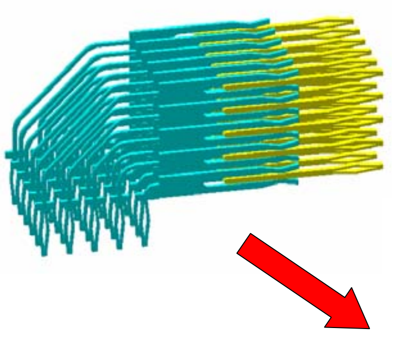
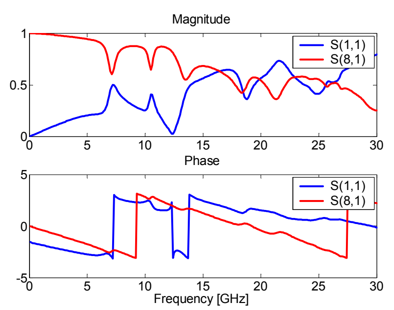
Possible scenarios



Frequency-Domain
full-wave simulation
(MoM, FEM, …)



Frequency tables
of transfer matrix
(S, Y, Z, …)

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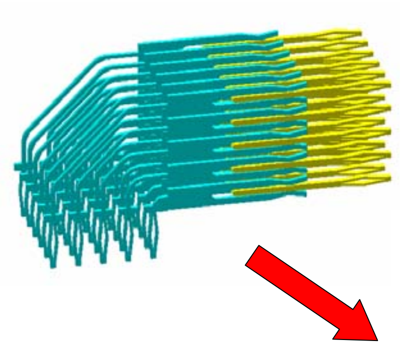
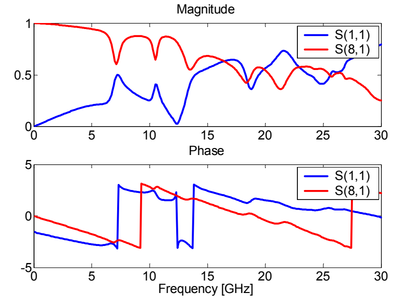
Possible scenarios



Time-Domain full-wave simulation (FIT, FDTD)

FFT postprocessing



Frequency tables of transfer matrix (S, Y, Z, …)

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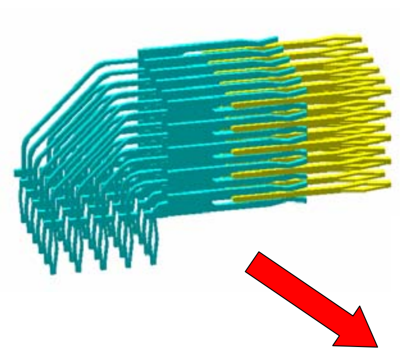
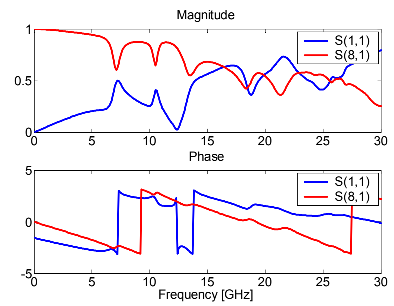
Possible scenarios



Direct VNA measurement



Frequency tables of transfer matrix (S)

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Frequency-Domain Macromodeling

Input data

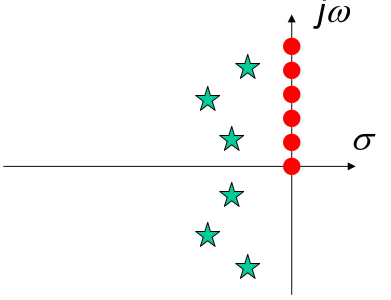
$$\left\{ \hat{H}(j\omega_k), \quad k = 1, \ldots, K \right\}$$

Approximation

$$H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty$$

Fitting condition

$$H(j\omega_k) \approx \hat{H}(j\omega_k), \forall k$$



Unknowns:

- Poles p_n
- Residues R_n
- Constant H_∞

Frequency-Domain Macromodeling

Direct fitting condition: nonlinear!

$$\sum_{n=1}^{N} \frac{R_n}{j\omega_k - p_n} + H_\infty \approx \hat{H}(j\omega_k), \quad \forall k$$

- Nonlinear dependence on poles
- Requires nonlinear optimization (e.g. nonlinear least squares)
- Convergence problems (local minima, etc…)

Frequency-Domain Macromodeling

Direct fitting condition: nonlinear!

$$\sum_{n=1}^{N} \frac{R_n}{j\omega_k - p_n} + H_\infty \approx \hat{H}(j\omega_k), \quad \forall k$$

Vector Fitting avoids nonlinear optimization

B. Gustavsen, A. Semlyen, ``Rational approximation of frequency responses by **vector fitting**'', *IEEE Trans. Power Delivery*, Vol.14, July 1999, pp.1052-1061

Frequency-Domain Vector Fitting

Input data

$$\left\{ \hat{H}(j\omega_k), \quad k = 1,\ldots,K \right\}$$

Approximation

$$H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty$$

Weight function

$$w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1$$

- $w(s)$ is unitary for $s \to \infty$
- poles q_n are fixed a priori
- residues c_n are unknown

Vector Fitting condition

$$w(s)\,H(s) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$

The poles of $w(s)\,H(s)$ are $\{ q_n \}$ only!

Frequency-Domain Vector Fitting

Input data

$$\left\{ \hat{H}(j\omega_k), \quad k = 1,\ldots,K \right\}$$

Approximation

$$H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty$$

Weight function

$$w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1$$

$$= \prod_{n=1}^{N} \frac{(s - z_n)}{(s - q_n)}$$

There are N zeros $\{ z_n \}$

Vector Fitting condition

$$w(s)\, H(s) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$

$$\{p_n\} \cong \{z_n\}$$

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Frequency-Domain Vector Fitting

Input data

$$\left\{ \hat{H}(j\omega_k), \quad k = 1,\ldots,K \right\}$$

Weight function

$$w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1$$

$$\left\{ \sum_{n=1}^{N} \frac{c_n}{j\omega_k - q_n} + 1 \right\} \hat{H}(j\omega_k) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{j\omega_k - q_n} + \tilde{H}_\infty$$

Vector Fitting condition

$$w(s)\, H(s) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$

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Frequency-Domain Vector Fitting

Unknowns

$$\{c_n, \tilde{c}_n, \tilde{H}_\infty\}$$

$$\left\{ \sum_{n=1}^{N} \frac{c_n}{j\omega_k - q_n} + 1 \right\} \hat{H}(j\omega_k) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{j\omega_k - q_n} + \tilde{H}_\infty$$

Linear least squares problem: easy to solve!

$$w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1 = \prod_{n=1}^{N} \frac{(s - z_n)}{(s - q_n)}$$

Poles of $H(s)$

Frequency-Domain Vector Fitting

$$w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1 = \prod_{n=1}^{N} \frac{(s - z_n)}{(s - q_n)}$$

Poles of $H(s)$

Theorem: the zeros $\{ z_n \}$ are the eigenvalues of

$$\mathbf{Q} = \mathbf{A} - \mathbf{b}\,\mathbf{c}^T$$

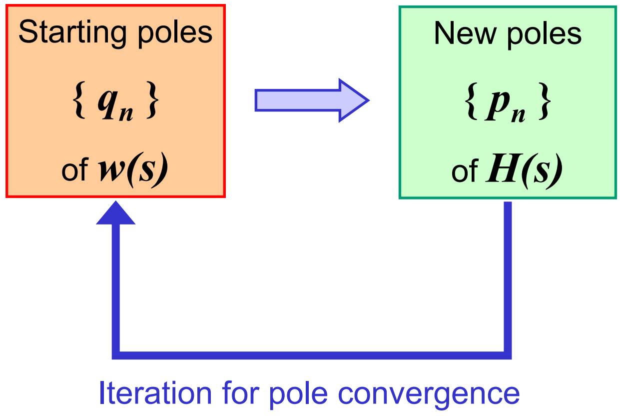
where

$$\mathbf{A} = \text{diag}\,\{ q_n \}$$

$$\mathbf{b} = (1 \quad 1 \quad \cdots \quad 1)^T$$

$$\mathbf{c} = (c_1 \quad c_2 \quad \cdots \quad c_N)^T$$

Pole relocation

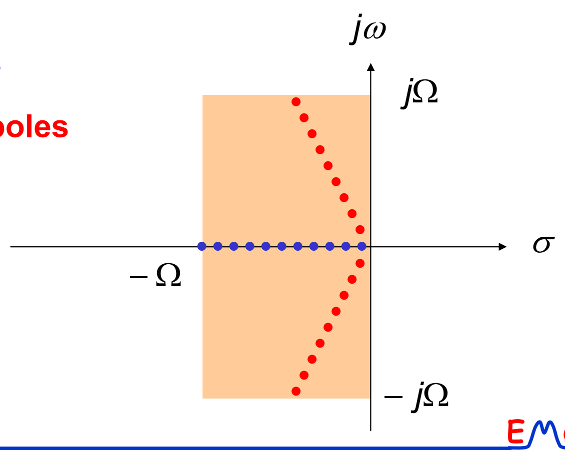


Choice of starting poles

Uniform coverage of approximation bandwidth

Real poles

Complex poles



Vector Fitting: residues

Input data

$$\left\{ \hat{H}(j\omega_k), \quad k = 1,\ldots,K \right\}$$

Approximation

$$H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty$$

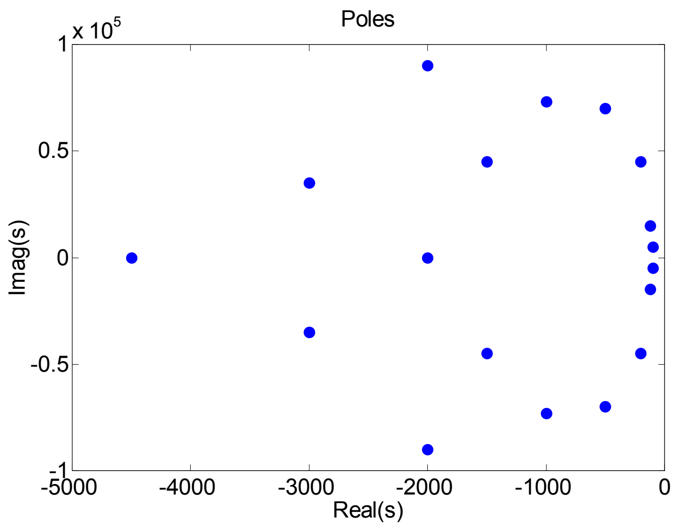
Fitting condition

$$H(j\omega_k) \approx \hat{H}(j\omega_k), \forall k$$

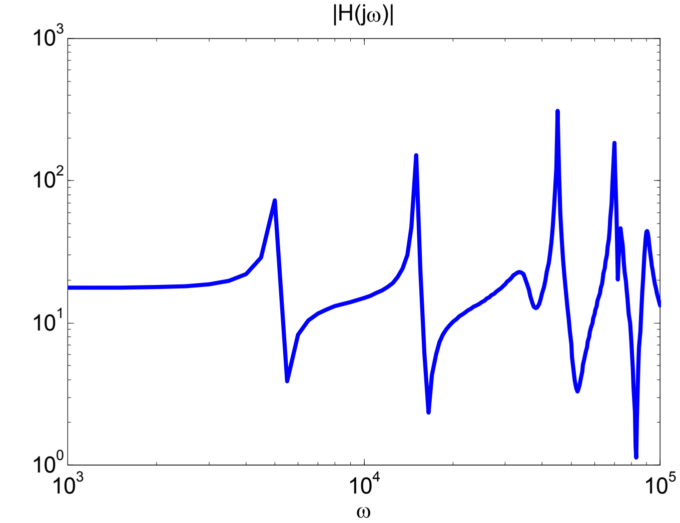
Another linear least squares problem!

Residues R_n and constant H_∞

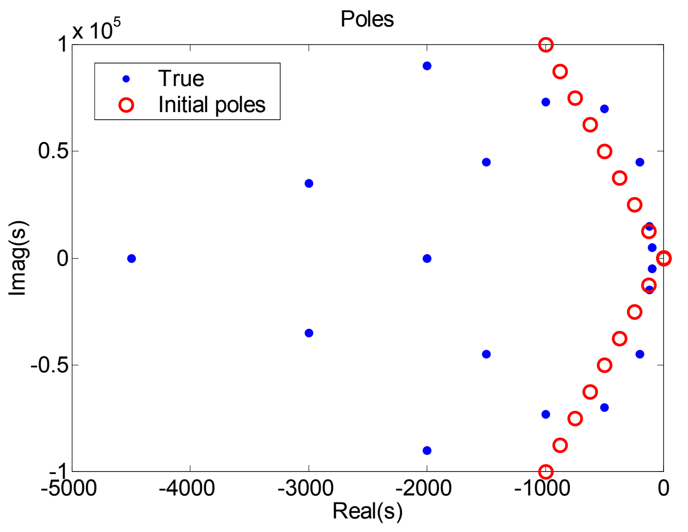
Example 1: 18 random poles/residues



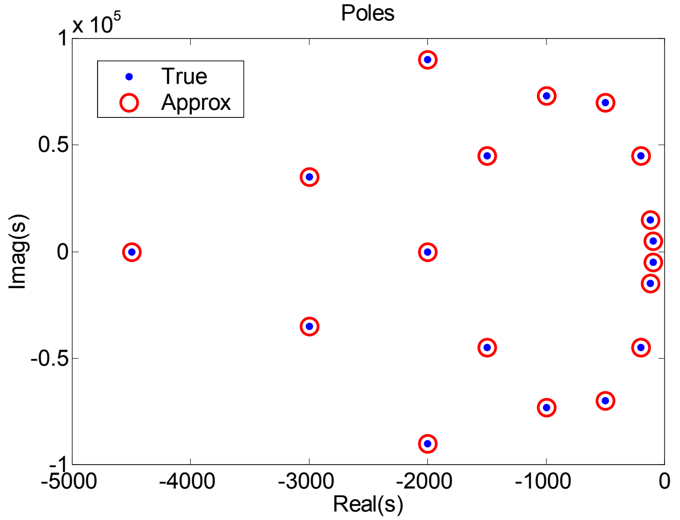
Example 1



Example 1



Example 1



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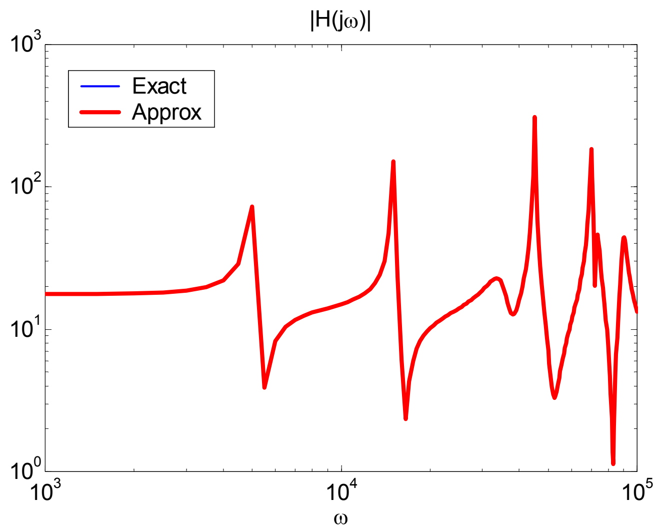
Example 1



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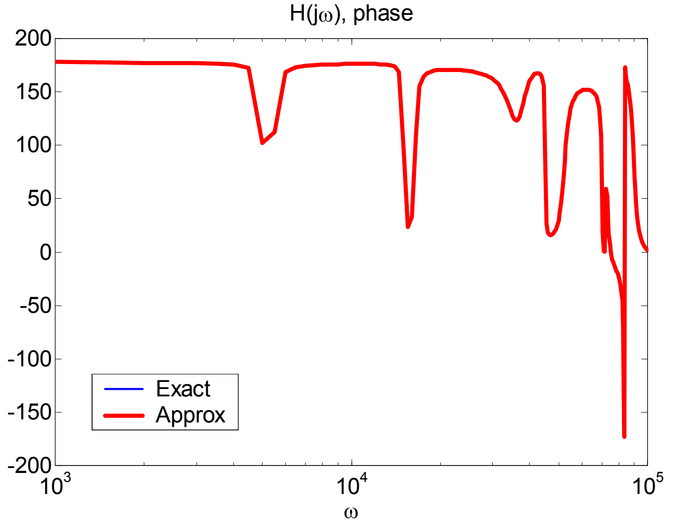
Example 1

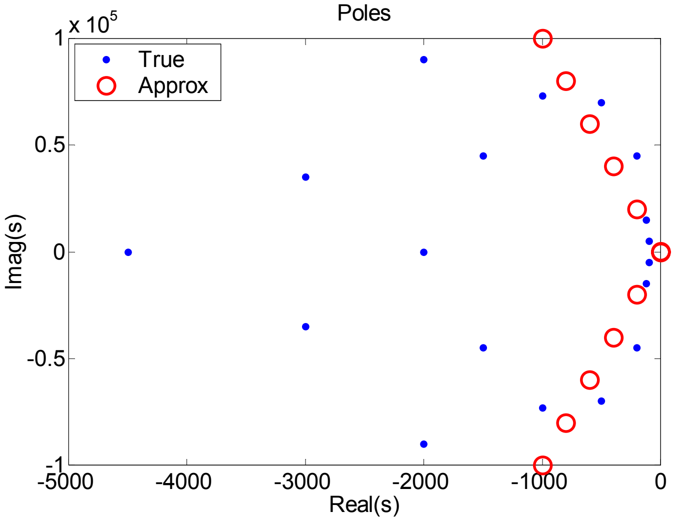


Example 2

Same 18-pole rational function

Reduced-order fitting (12th order)

Example 2



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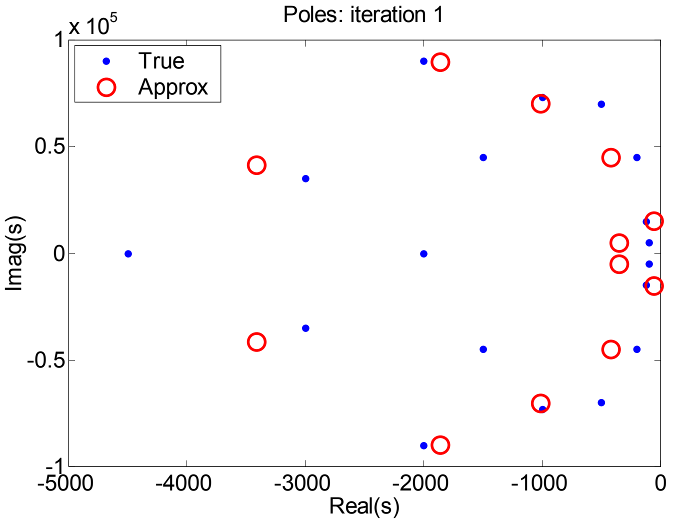
Example 2



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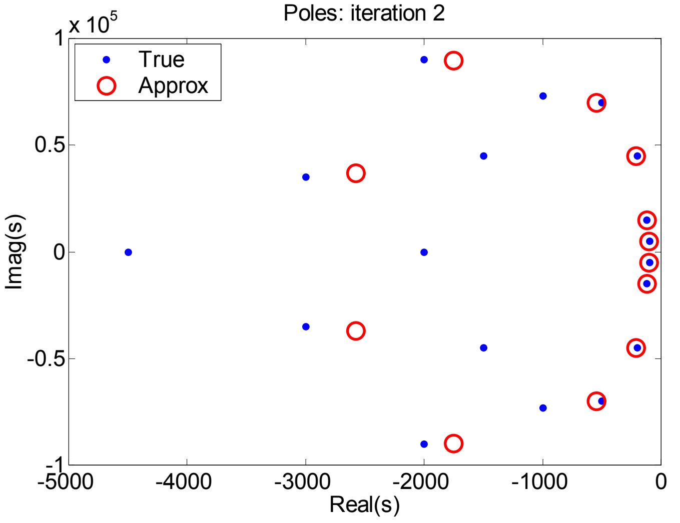
Example 2



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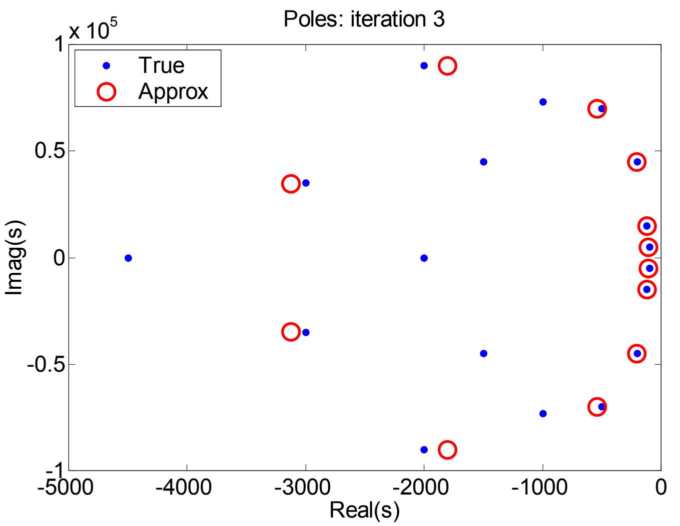

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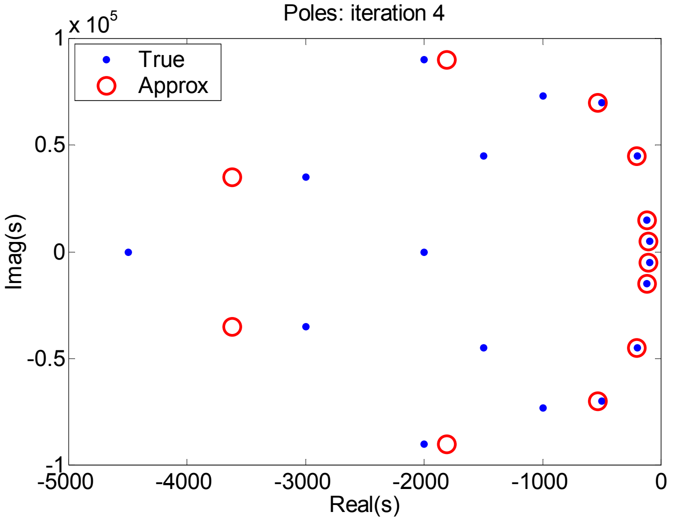

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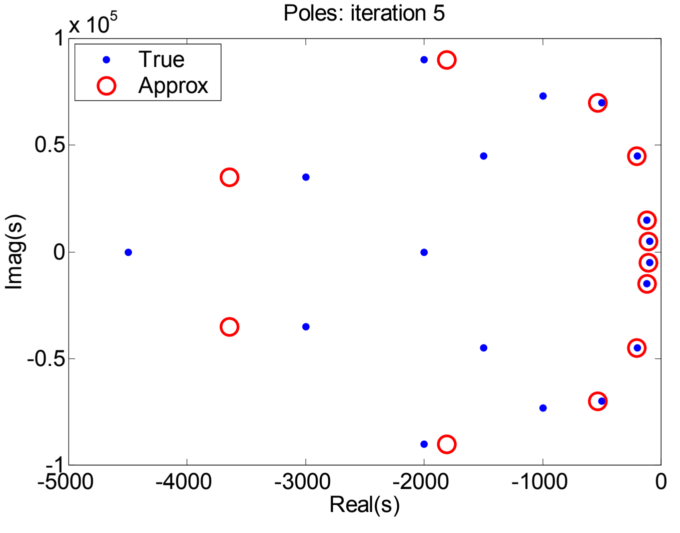
Example 2



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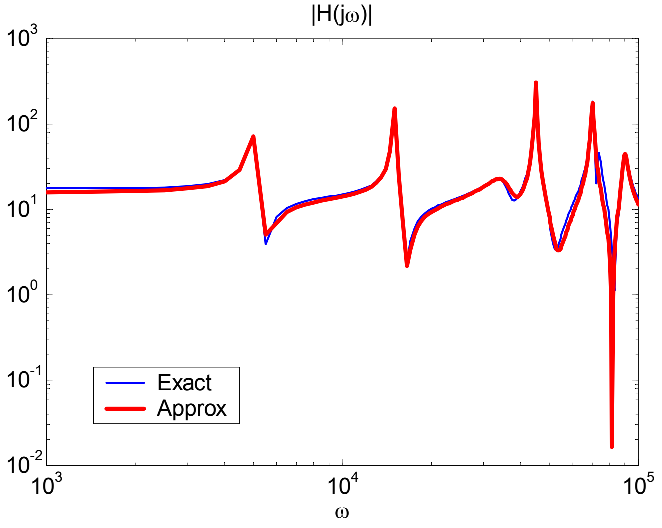
Example 2



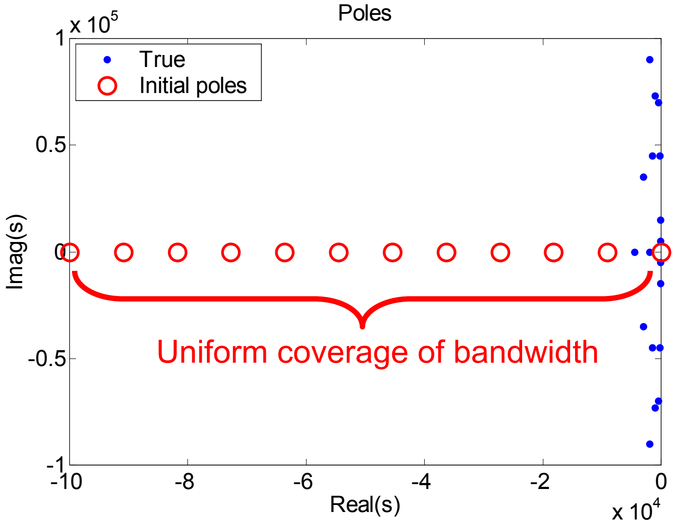
Example 3

Same 18-pole rational function

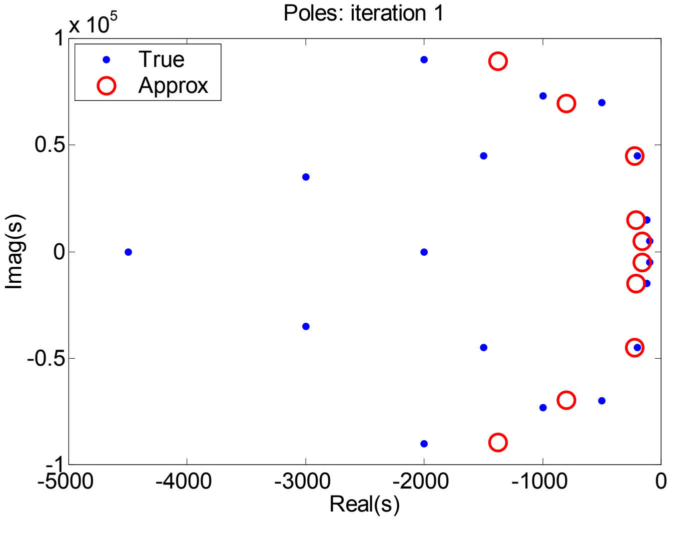
Reduced-order fitting (12th order)

Different starting poles (real poles)

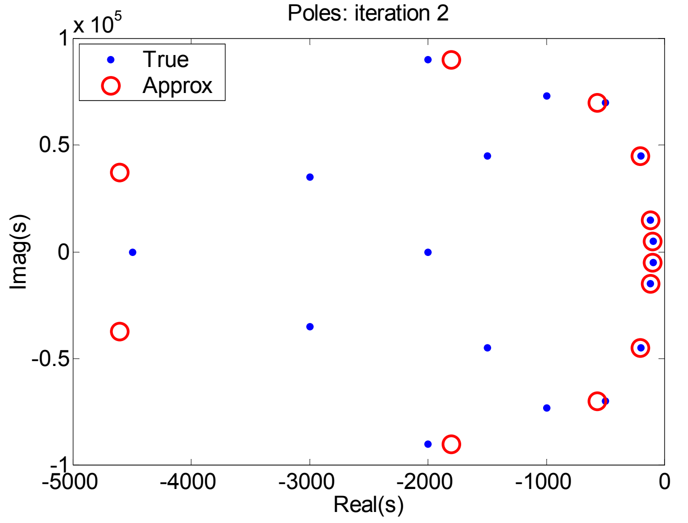
Example 3



Example 3



Example 3



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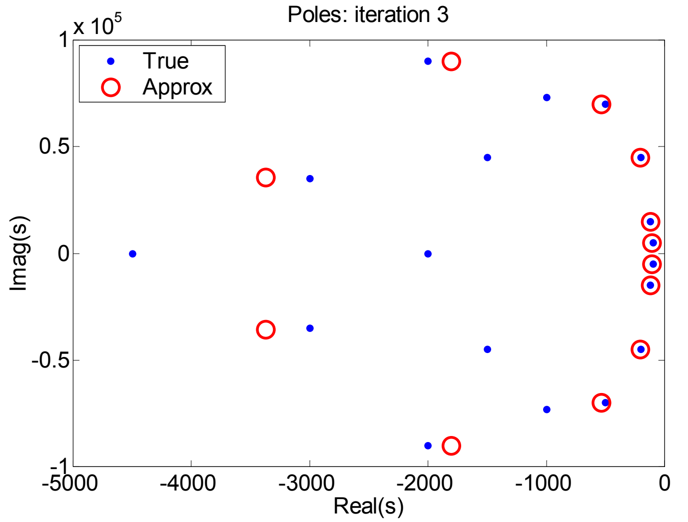
Example 3



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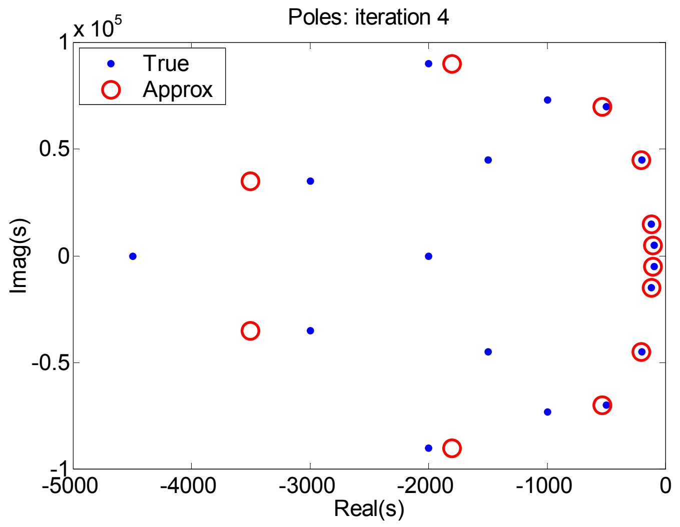
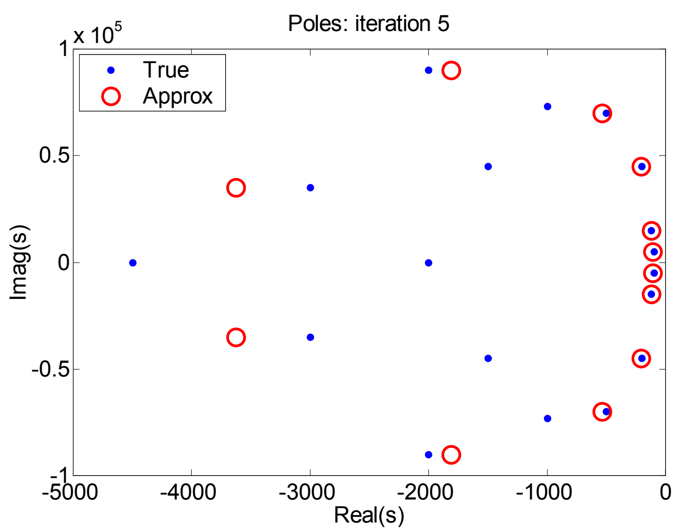
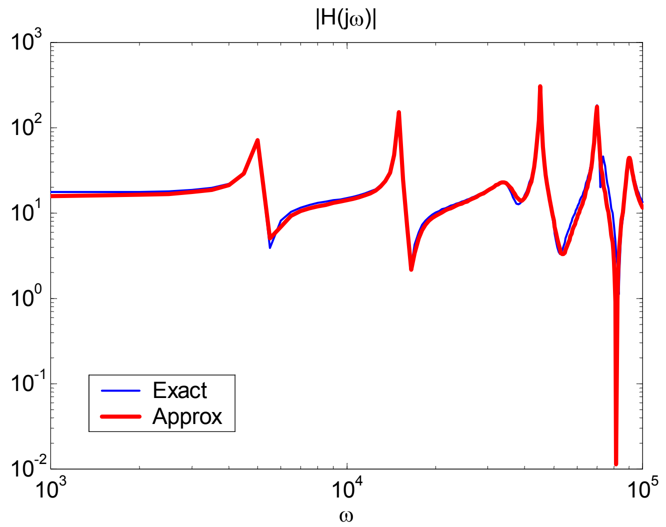
Example 3



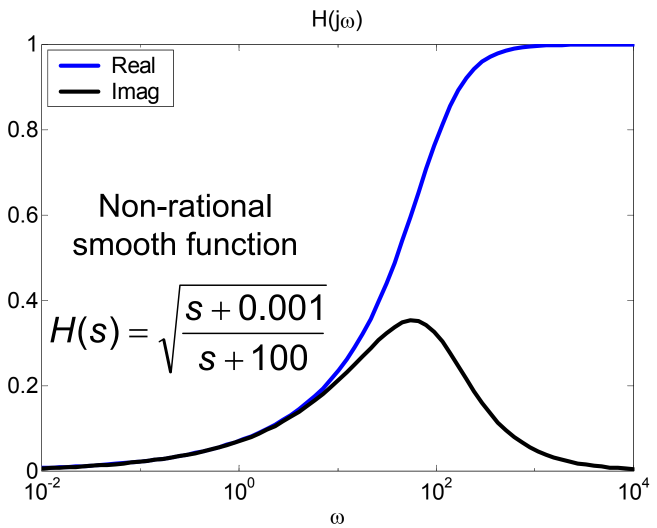
Example 3



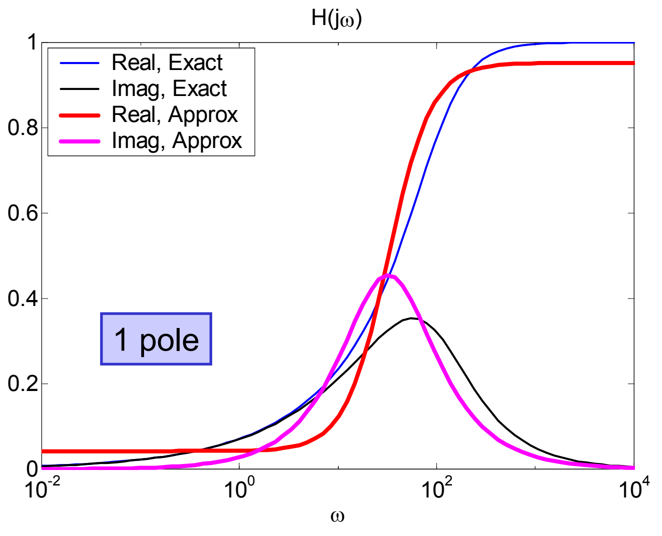
Example 3



Example 4



Example 4



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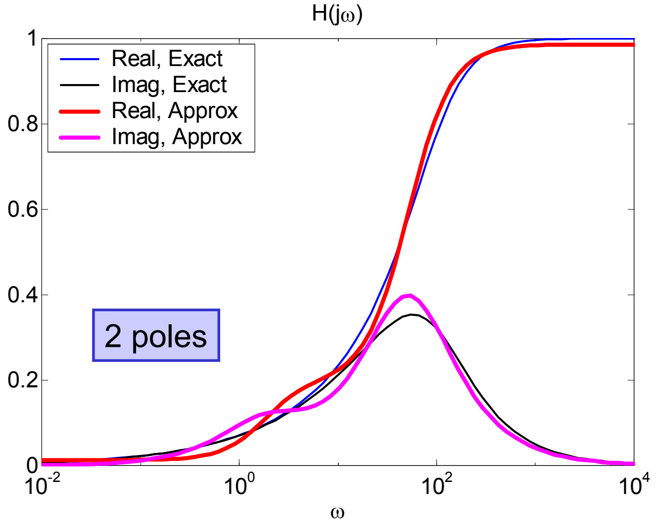
Example 4



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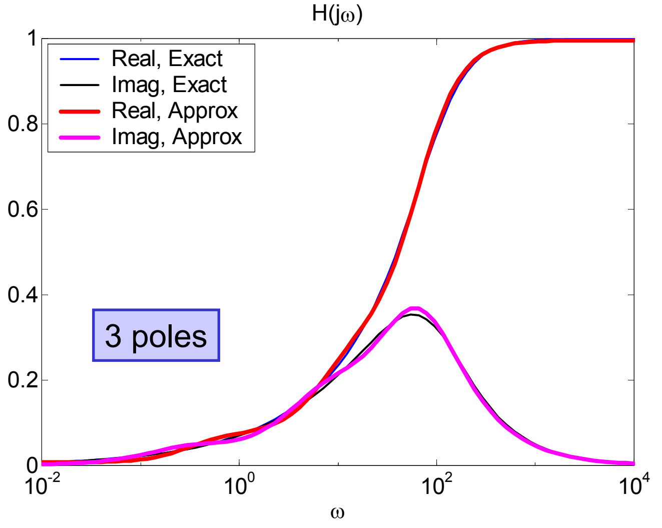
Example 4



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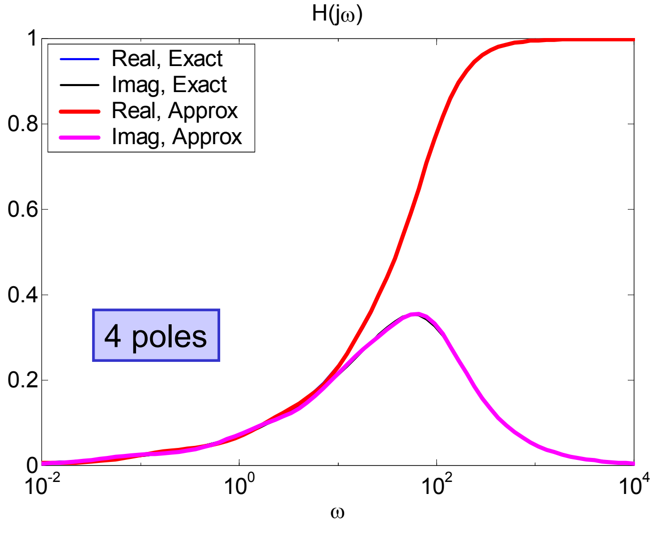
Example 4



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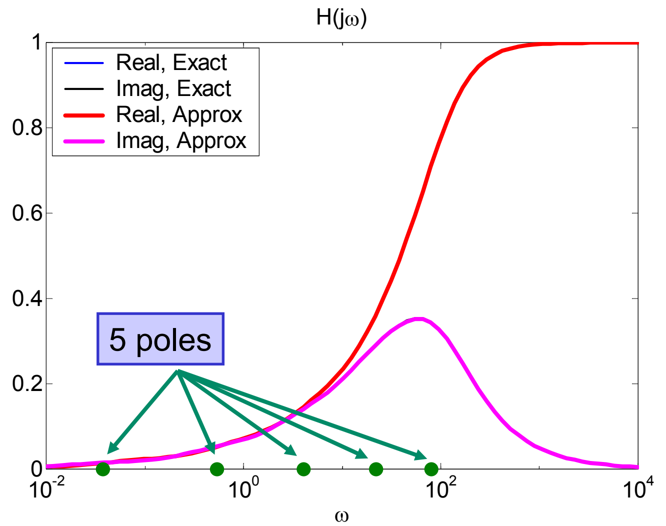
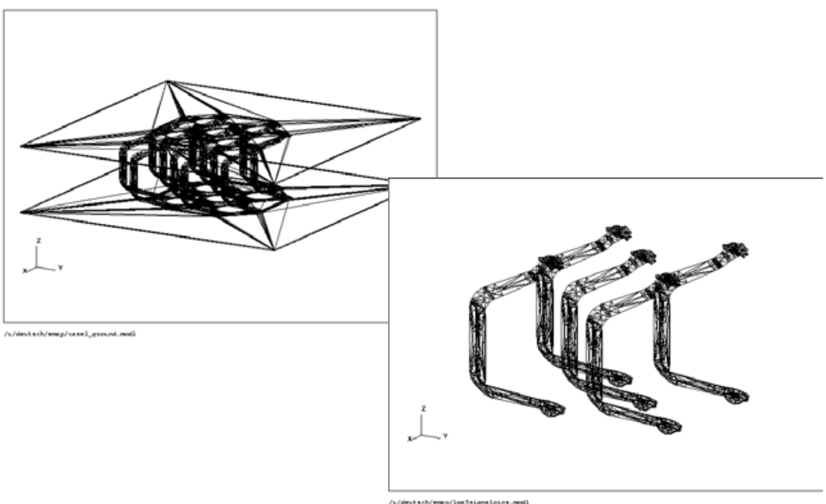
Example 4

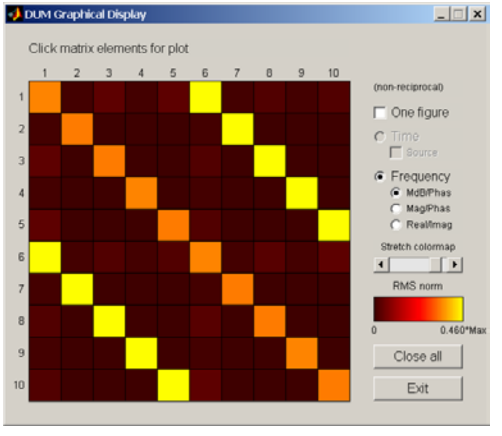


Example 5: MCM-board connector



Example 5: MCM-board connector

Data: 10-port structure, frequency-domain S-matrix



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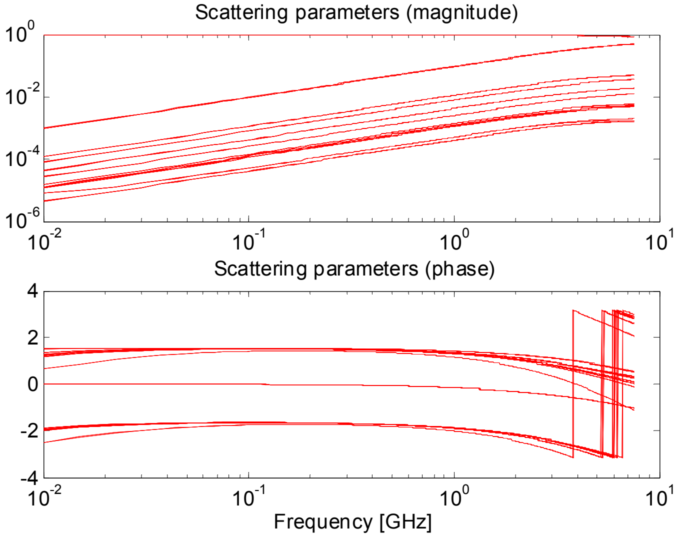
Example 5: MCM-board connector



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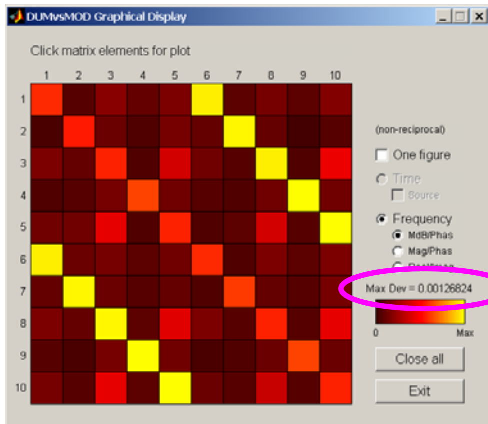
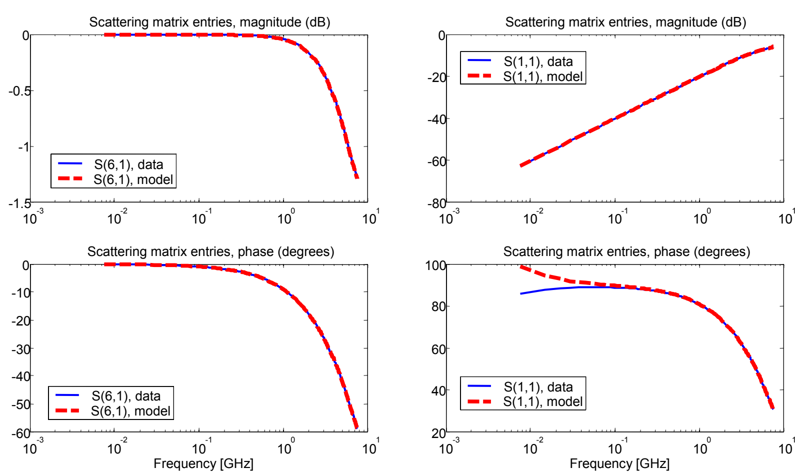
Example 5: MCM-board connector

Macromodel: 4-poles



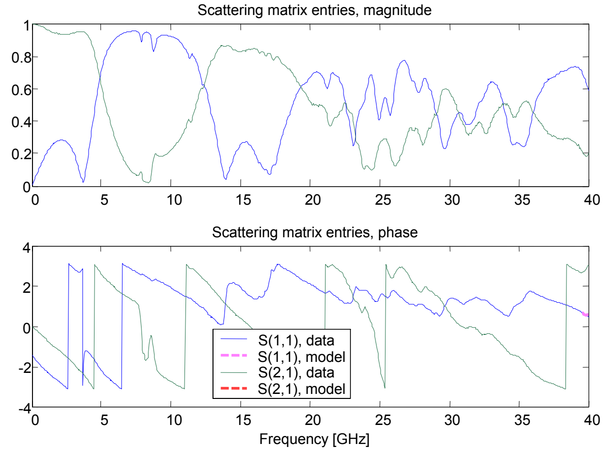
Error: 0.1%

Example 5: MCM-board connector



Example 6: stripline+lauches

Data: measured S-parameters



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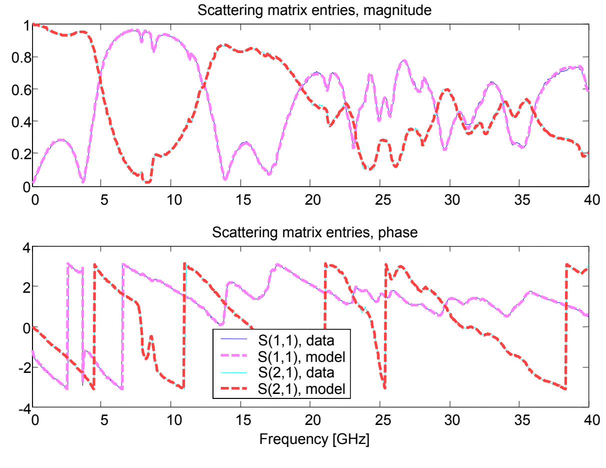
Example 6: stripline+lauches

Macromodel: 60 poles



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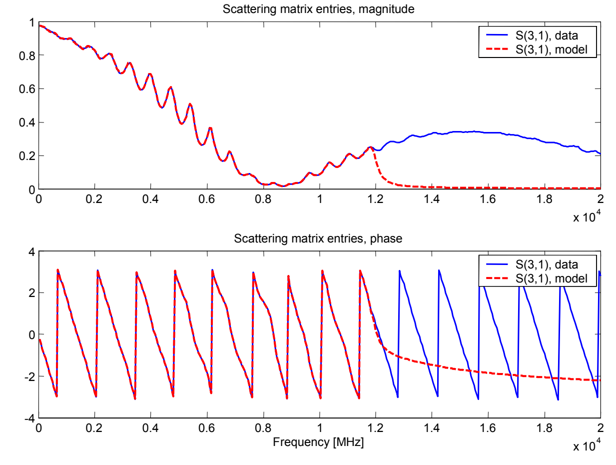
Example 7: PCB path, measured

VF with frequency-selective weighting



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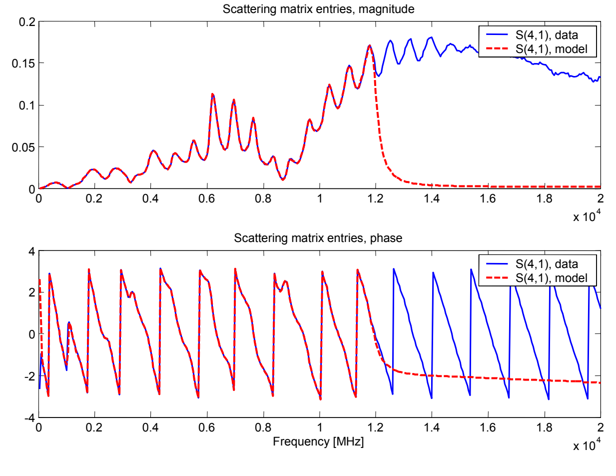
Example 7: PCB path, measured

VF with frequency-selective weighting



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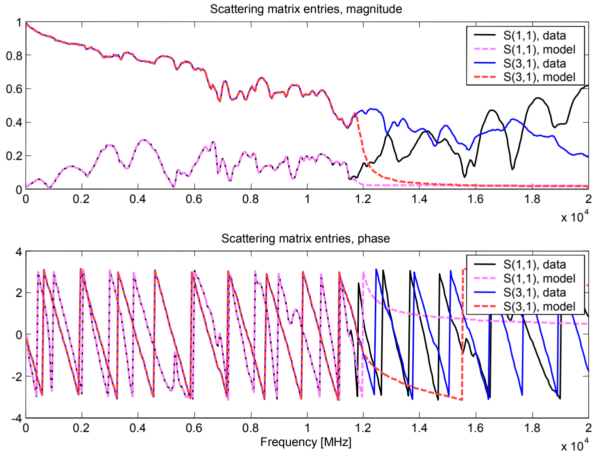
Example 8: Connector, measured

VF with frequency-selective weighting



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Vector Fitting: summary

- Tool for frequency-domain rational approximation
  - rational transfer functions (system identification)
  - rational transfer functions (reduced-order modeling)
  - non-rational transfer functions
- **Data from full-wave simulations**
- **Direct frequency-domain measurements**

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Vector Fitting: summary

www.energy.sintef.no/produkt/VECTFIT/home.asp

- Very accurate and robust
- Only linear least squares + eigenvalues required
- Stability is not guaranteed
  - can be fixed by flipping real part during relocation
- Passivity is not guaranteed
  - can be fixed a posteriori (see later)

Outline

- Introduction
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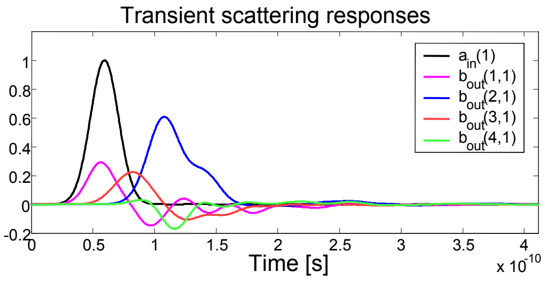
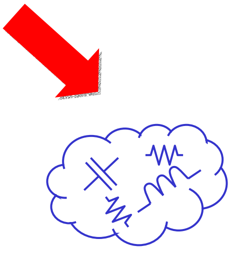
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

Time-domain macromodeling

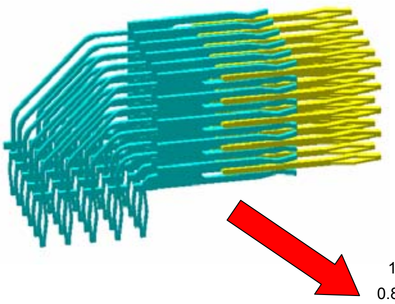
Model identification from time-domain responses



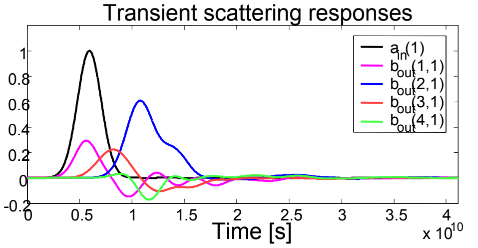
IDENTIFICATION



Possible scenarios

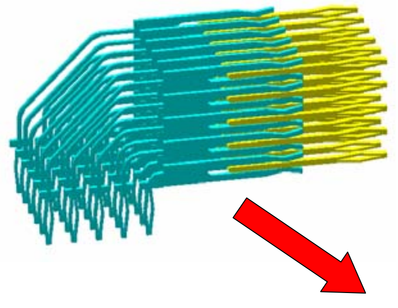


Time-Domain full-wave simulation (FIT, FDTD)



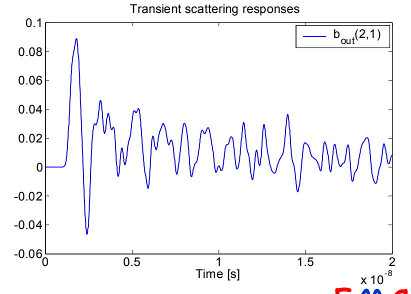
Port responses to transient excitations (usually gaussian)

Possible scenarios



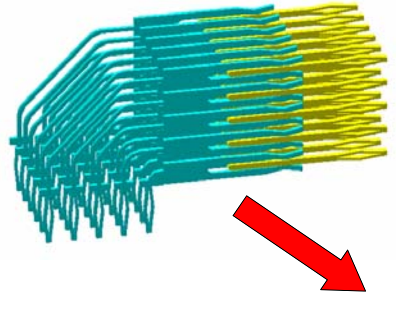
Time-Domain full-wave simulation (FIT, FDTD)

Truncated waveforms from short FDTD runs



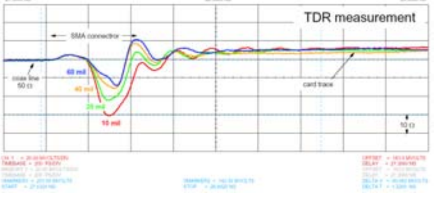
Port responses to transient excitations (usually gaussian)

Possible scenarios



Time-domain measurements

(work in progress)



Port responses to transient excitations (usually gaussian)

Time-Domain Macromodeling

Input pulse

$\mathbf{x}(t)$ t – domain

Output responses

$\mathbf{y}(t)$ t – domain

Transfer function

$$\mathbf{Y}(s) = \mathbf{H}(s)\mathbf{X}(s)$$

Rational approximation

$$\mathbf{H}(s) \approx \mathbf{H}_\infty + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

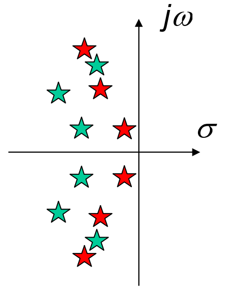
Unknowns:
- Poles p_n
- Residues $\mathbf{R}_n$
- Constant $\mathbf{H}_\infty$

Time-Domain Vector Fitting

Step 1. Find the dominant poles via "relocation"

Guess poles $\{ q_n \}$ → New poles $\{ p_n \}$

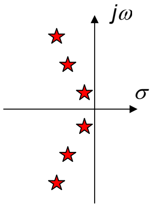
Iterative refinement



How to do it using time-domain data?

How to insure convergence to the right poles?

Time-Domain Vector Fitting



1a. Start with initial poles: $\{ q_n \}$

1b. Define weight function: unknown $\{ k_n \}$

$$w(s) = 1 + \sum_n \frac{k_n}{s - \boxed{q_n}}$$

Starting poles

1c. Assume the following condition

$$w(s)\mathbf{H}(s) = a + \sum_n \frac{b_n}{s - \boxed{q_n}}$$

Poles of $\mathbf{H}(s)$ = Zeros of $w(s)$

Time-Domain Vector Fitting

$$w(s)\mathbf{H}(s) = a + \sum_n \frac{b_n}{s - q_n}$$

Apply the input pulse $\mathbf{X}(s)$

$$w(s)\mathbf{Y}(s) = \left(a + \sum_n \frac{b_n}{s - q_n} \right) \mathbf{X}(s)$$

Compute inverse Laplace transform

$$\mathbf{y}(t) + \sum_n k_n \, \mathbf{y}_n(t) = a \, \mathbf{x}(t) + \sum_n b_n \, \mathbf{x}_n(t)$$

$$\mathbf{x}_n(t) = \int_0^t e^{q_n (t-\tau)} \mathbf{x}(\tau) d\tau$$

$$\mathbf{y}_n(t) = \int_0^t e^{q_n (t-\tau)} \mathbf{y}(\tau) d\tau$$

Low-pass filtered input and output signals

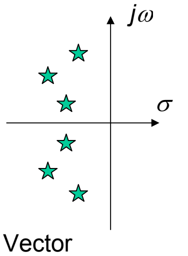
Time-Domain Vector Fitting

1d. Solve a linear least squares system for $\boxed{k_n}$, a , b_n

$$\boxed{\mathbf{y}(t) + \sum_n k_n \, \mathbf{y}_n(t) = a \, \mathbf{x}(t) + \sum_n b_n \, \mathbf{x}_n(t)}$$

1e. Compute the zeros $\{ p_n \}$ of the weight function

$$w(s) = 1 + \sum_n \frac{\boxed{k_n}}{s - q_n} = \frac{\prod_n (s - \boxed{p_n})}{\prod_n (s - q_n)}$$



These are the dominant poles!

S. Grivet-Talocia, "Package Macromodeling via Time-Domain Vector Fitting", *IEEE Microwave Wireless Comp. Lett.*, Nov. 2003

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Time-Domain Vector Fitting

Step 2. Compute the residues

2a. Low-pass filter input signals with new poles

$$\boxed{\tilde{\mathbf{x}}_n(t) = \int_0^t e^{p_n (t-\tau)} \mathbf{x}(\tau) d\tau}$$

2b. Solve a linear least squares system for $\mathbf{R}_n$ and $\mathbf{H}_\infty$

$$\boxed{\mathbf{y}(t) = \mathbf{H}_\infty \, \mathbf{x}(t) + \sum_n \mathbf{R}_n \, \tilde{\mathbf{x}}_n(t)}$$

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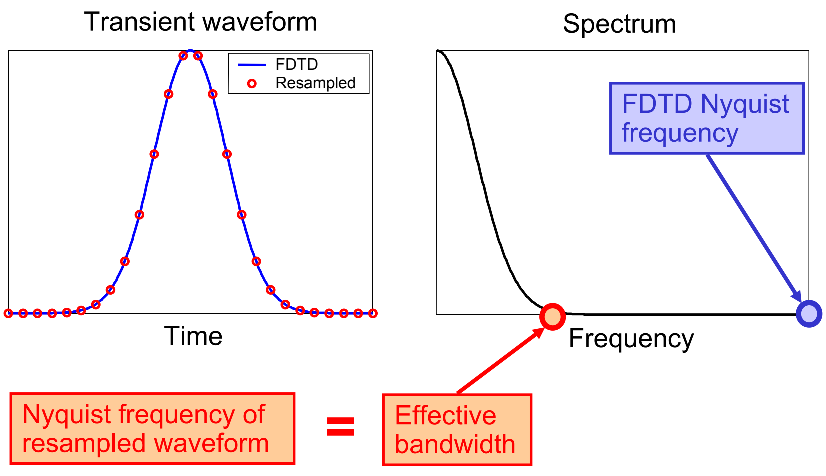
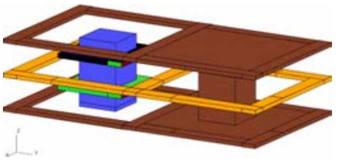
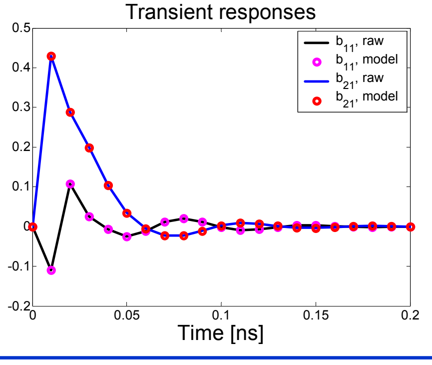
Subsampling



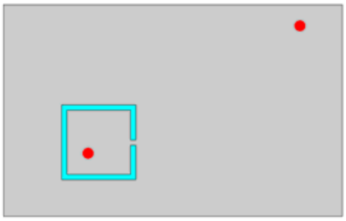
Example 1: single via



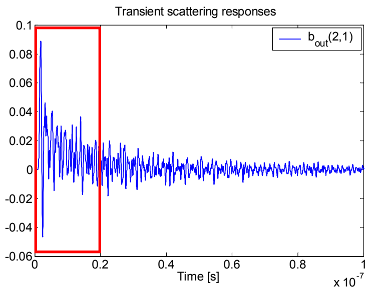
Raw data:
Triangle Impulse Responses obtained by a transient PEEC solver (by Dr. Ruehli, IBM)

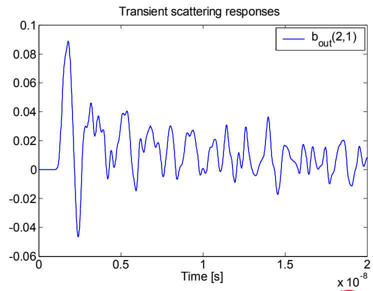


Example 2: segmented power bus



- 2-port structure
- Time-Domain solution
- CST Microwave Studio
- Bandwidth: 3 GHz
- 50 Ω port terminations





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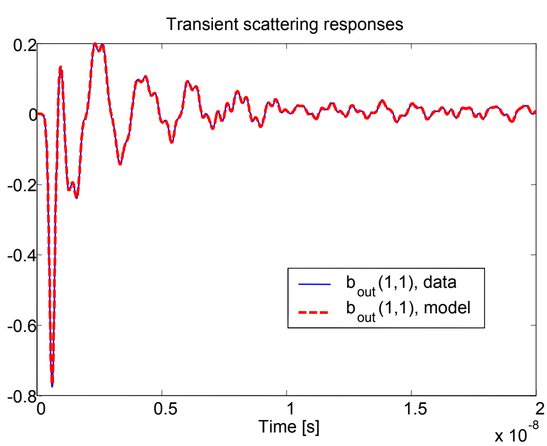
Example 2: segmented power bus

80-poles model (Time-Domain Vector Fitting)



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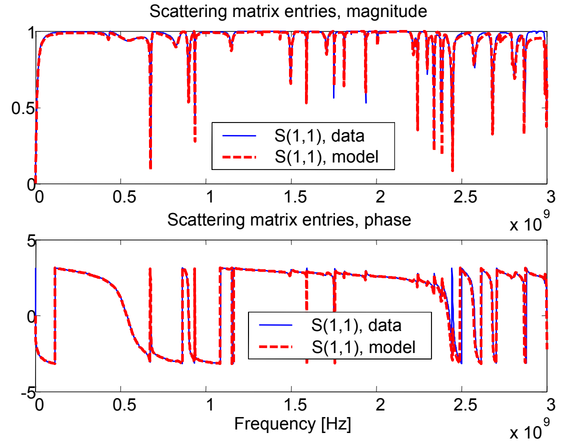
Example 2: segmented power bus

Comparison vs. frequency-domain scattering data



Example 2: segmented power bus

Full-wave simulation time (CST) to compute…

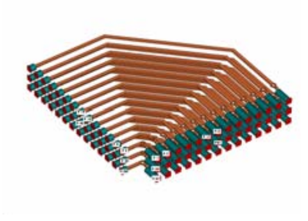
… frequency scattering data: 60 hours

(wait until transients are finished for reliable FFT)

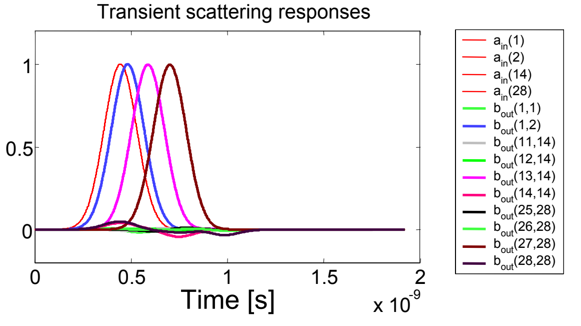
… macromodel: 6 hours

(can use truncated waveforms for TD-VF)

Example 3: 42-pin connector



3x14 pins, 84 ports
Characterized via FIT
(CST Microwave Studio 4)
(Courtesy: Erni - AdMOS)



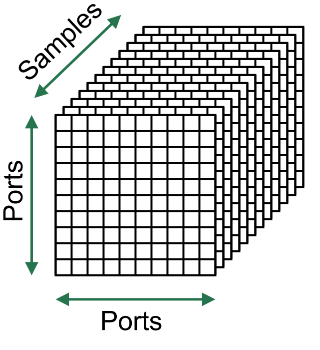
Handling many ports

Frequency-Domain Vector Fitting

$$\left(1 + \sum_n \frac{k_n}{s - q_n}\right) \mathbf{H}(s) = a + \sum_n \frac{b_n}{s - q_n}$$

Time-Domain Vector Fitting

$$\mathbf{y}(t) + \sum_n k_n \, \mathbf{y}_n(t) = a \, \mathbf{x}(t) + \sum_n b_n \, \mathbf{x}_n(t)$$



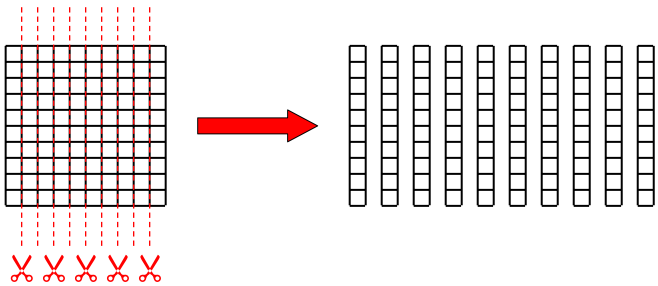
Processing **all** responses may lead to a **large** system!

Handling many ports

1. Split port responses into subsets

Transfer matrix $\mathbf{H}(s)$

Subsets $\{\mathbf{h}_k(s)\}$



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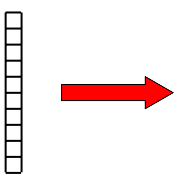
Handling many ports

2. Macromodel each subset via FD-VF or TD-VF



$$\mathbf{h}_k(s) \approx \mathbf{h}_{k,\infty} + \sum_n \frac{\mathbf{r}_{k,n}}{s - p_{k,n}}$$

Partial state-space representation

$$\begin{cases} \dot{\mathbf{w}}_k = \mathbf{A}_k \mathbf{w}_k + \mathbf{B}_k \mathbf{x}_k \\ \mathbf{y}_k = \mathbf{C}_k \mathbf{w}_k + \mathbf{D}_k \mathbf{x}_k \end{cases}$$

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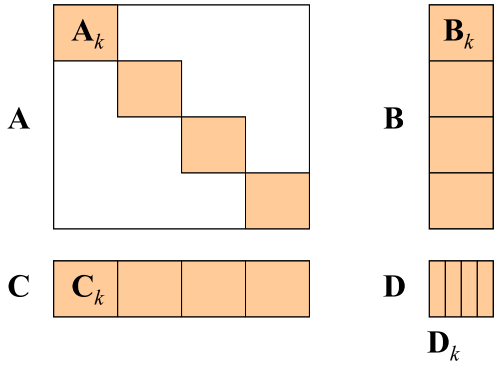
Handling many ports

4. Assemble all partial models into a global model

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{B}\;\mathbf{x} \\ \mathbf{y} = \mathbf{C}\,\mathbf{w} + \mathbf{D}\;\mathbf{x} \end{cases}$$

All matrices can be constructed as sparse!

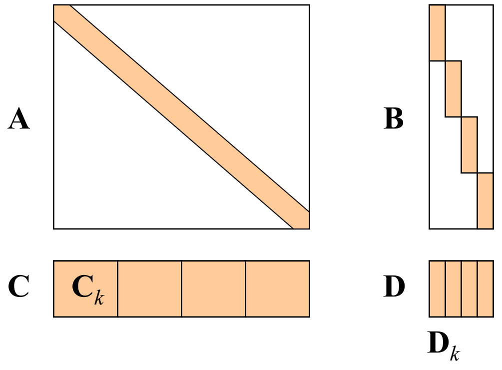


Handling many ports

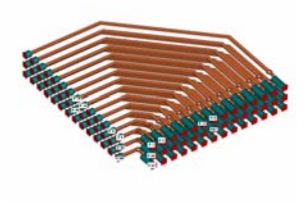
4. Assemble all partial models into a global model

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{B}\;\mathbf{x} \\ \mathbf{y} = \mathbf{C}\,\mathbf{w} + \mathbf{D}\;\mathbf{x} \end{cases}$$

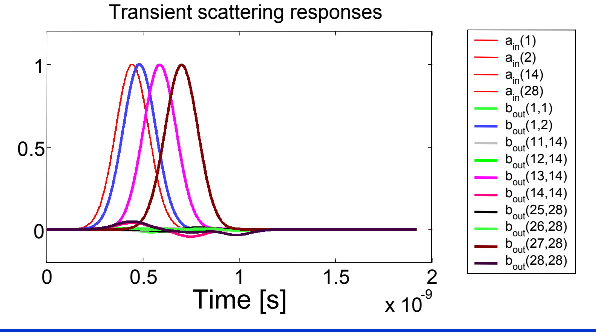
All matrices can be constructed as sparse!



Example 3: 42-pin connector

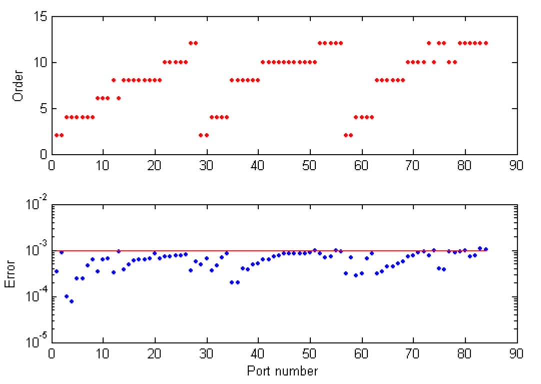


3x14 pins, 84 ports
Characterized via FIT
(CST Microwave Studio 4)
(Courtesy: Erni - AdMOS)

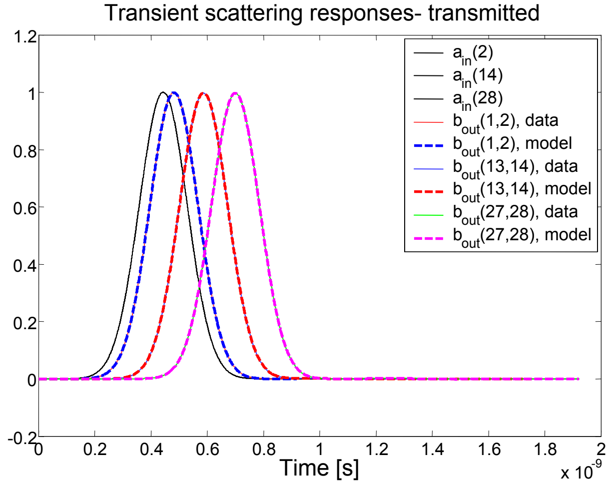


Example 3: model order selection

Automatic (iterative) order selection on each of the 84 subsets of port responses (reduced model complexity)



Example 3: macromodel accuracy



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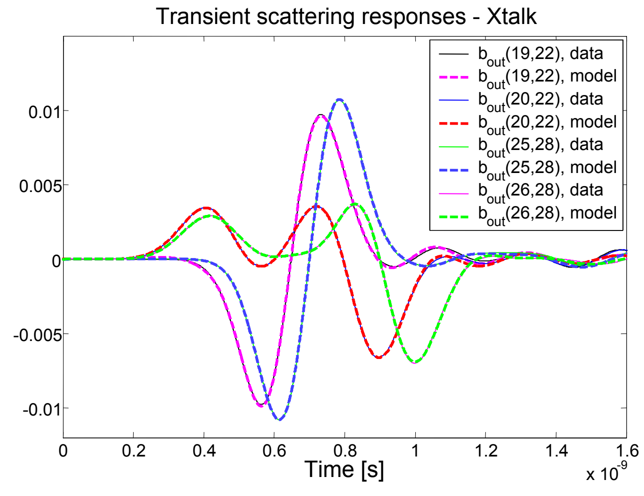
Example 3: macromodel accuracy



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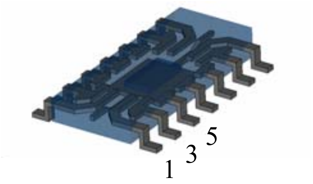
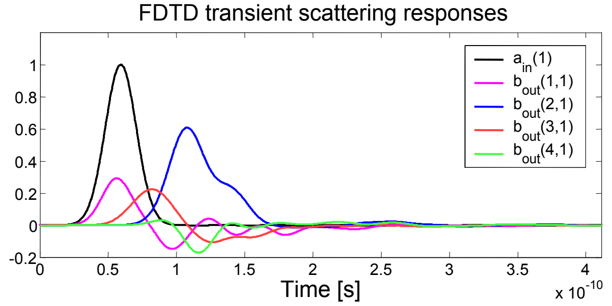
Example 4: 14-pin package



14-pin SOIC package
Simplified CAD for FDTD
Bandwidth: 40 GHz
50 Ω port terminations



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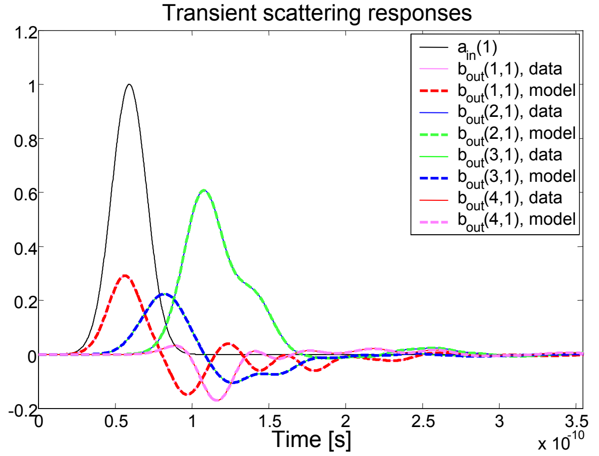
Example 4: macromodel responses



No visible difference between data and model

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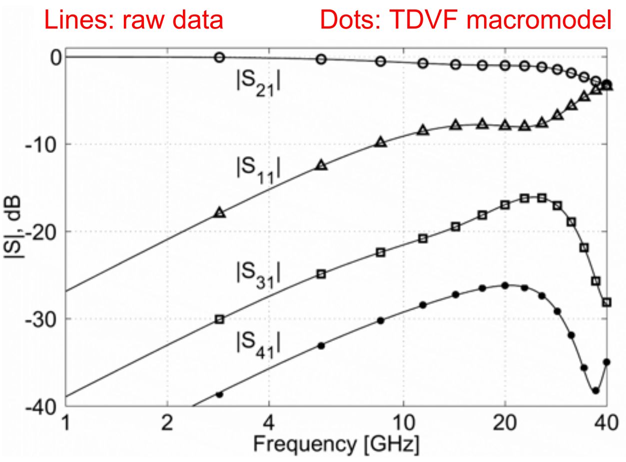
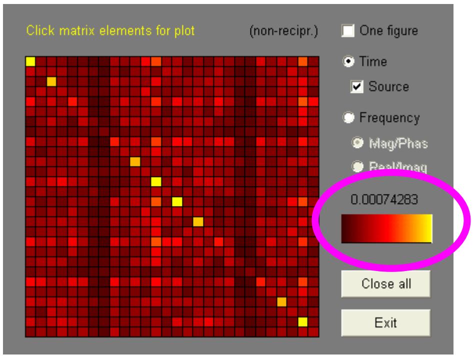
Example 4: macromodel responses



No visible difference between data and model

Example 4: macromodel accuracy

Maximum deviation between model and data for all 28x28 responses



Largest: 0.00074

TD-VF produces highly accurate macromodels

Macromodel properties

☺ **Accuracy**

Good initial data $\Rightarrow$ small approximation errors

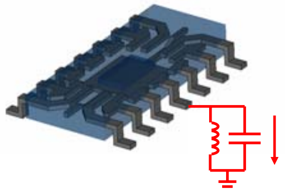
☺ **Stability**

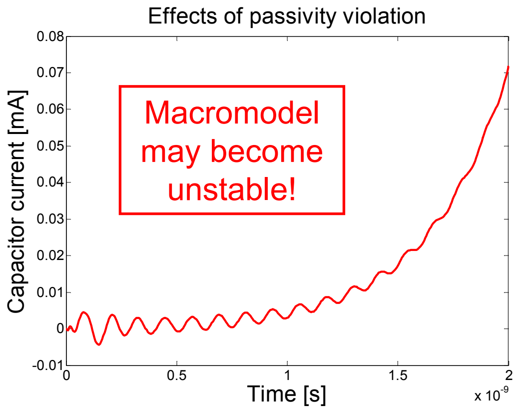
All poles with negative real part

☹ **Passivity**

The macromodel may not be passive

Example 4: change terminations





Port terminations:

$R = 50\ \mathrm{m}\Omega \div 50\ \Omega$

$L = 1\ \mathrm{nH}$

$C = 1\ \mathrm{pF}$

Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

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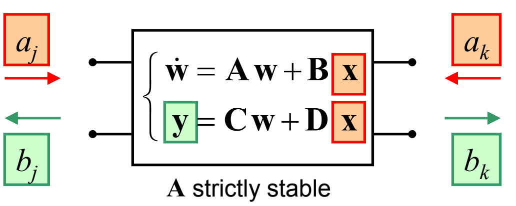
Passivity conditions

Scattering representation



$\mathbf{A}$ strictly stable

Scattering matrix: must be bounded real

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

$$\boxed{\{\text{singular values of } \mathbf{H}(j\omega)\} \leq \mathbf{1}, \quad \forall \omega}$$

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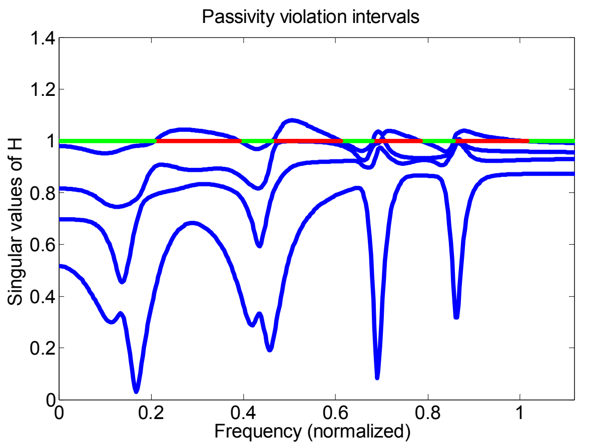
Passivity conditions

Scattering representation



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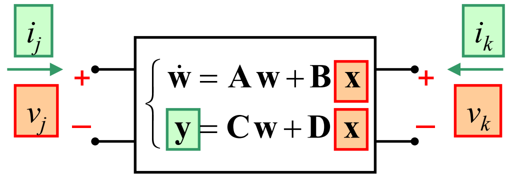
Passivity conditions

Admittance representation



$\mathbf{A}$ strictly stable

Admittance matrix: must be positive real

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

$$\boxed{\left\{\text{eigenvalues of } \left(\mathbf{H}(j\omega) + \mathbf{H}^H(j\omega)\right)\right\} \geq \mathbf{0}, \quad \forall \omega}$$

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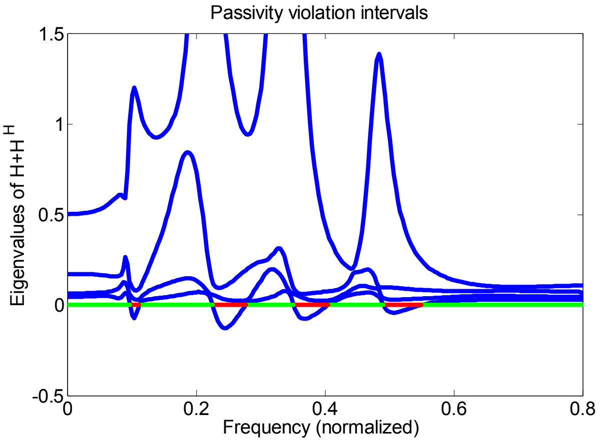
Passivity conditions

Admittance representation



Checking passivity

Scattering representation

$$\boxed{\{\text{singular values of } \mathbf{H}(j\omega)\} \le \mathbf{1}, \quad \forall \omega}$$

Several techniques can be used

Frequency sweep test: most straightforward

- Choose a set of frequency samples
- Compute $\mathbf{H}$ and its singular values, and check
- Time-consuming for large models
- May give wrong answers due to poor sampling

Checking passivity

Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq \mathbf{1}, \quad \forall \omega$$

Equivalent purely algebraic conditions:

- Linear Matrix Inequalities (**LMI**)
- Albegraic Riccati Equations (**ARE**)
- Eigenvalues of **Hamiltonian matrices**

S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan, “Linear Matrix Inequalities in System and Control Theory, SIAM, Philadelphia, 1994

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Checking passivity

Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq \mathbf{1}, \quad \forall \omega$$

Linear Matrix Inequality (LMI)

$$\begin{pmatrix} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{C}^T\mathbf{C} & \mathbf{P}\mathbf{B} + \mathbf{C}^T\mathbf{D} \\ \mathbf{B}^T\mathbf{P} + \mathbf{D}^T\mathbf{C} & \mathbf{D}^T\mathbf{D} - \mathbf{I} \end{pmatrix} \leq \mathbf{0} \qquad \mathbf{P} = \mathbf{P}^T, \mathbf{P} > 0$$

Real matrix $\mathbf{P}$ is the variable

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Checking passivity

Scattering representation

$$\boxed{\{\text{singular values of } \mathbf{H}(j\omega)\} \le \mathbf{1}, \quad \forall \omega}$$

Algebraic Riccati Equation (ARE)

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{C}^T\mathbf{C} + \left(\mathbf{P}\mathbf{B} + \mathbf{C}^T\mathbf{D}\right)\left(\mathbf{I} - \mathbf{D}^T\mathbf{D}\right)^{-1}\left(\mathbf{P}\mathbf{B} + \mathbf{C}^T\mathbf{D}\right)^T = \mathbf{0}$$

$$\mathbf{P} = \mathbf{P}^T$$

Real matrix $\mathbf{P}$ is the variable

Checking passivity

Scattering representation

$$\boxed{\{\text{singular values of } \mathbf{H}(j\omega)\} \le \mathbf{1}, \quad \forall \omega}$$

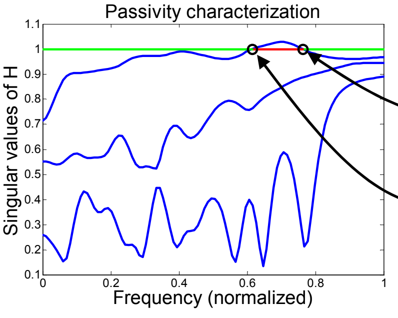
Eigenvalues of Hamiltonian matrix

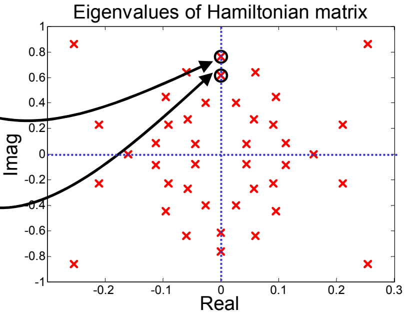
$$\mathbf{M} = \begin{pmatrix} \mathbf{A} - \mathbf{B}\left(\mathbf{D}^T\mathbf{D} - \mathbf{I}\right)^{-1}\mathbf{D}^T\mathbf{C} & -\mathbf{B}\left(\mathbf{D}^T\mathbf{D} - \mathbf{I}\right)^{-1}\mathbf{B}^T \\ \mathbf{C}^T\left(\mathbf{D}\mathbf{D}^T - \mathbf{I}\right)^{-1}\mathbf{C} & -\mathbf{A}^T + \mathbf{C}^T\mathbf{D}\left(\mathbf{D}^T\mathbf{D} - \mathbf{I}\right)^{-1}\mathbf{B}^T \end{pmatrix}$$

Real matrix $\mathbf{M}$ must have no imaginary eigenvalues

Checking passivity

Scattering representation



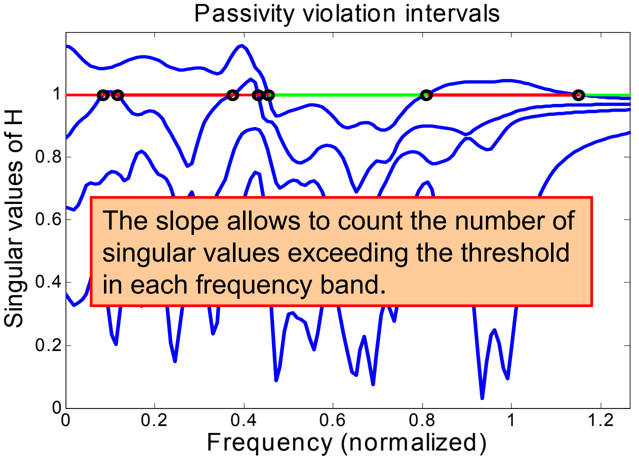


Theorem [Boyd, Balakrishnan, Kabamba, 1989]

$j\omega_0$ is an eigenvalue of $\mathbf{M} \Leftrightarrow \sigma = 1$ is a singular value of $\mathbf{H}(j\omega_0)$

Checking passivity

Scattering representation

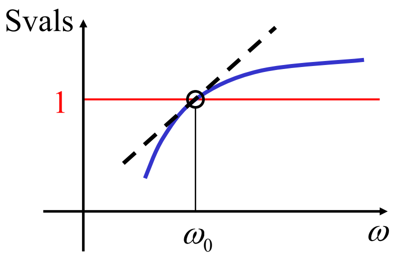


Checking passivity

Scattering representation

First-order perturbation of Hamiltonian eigenvalues

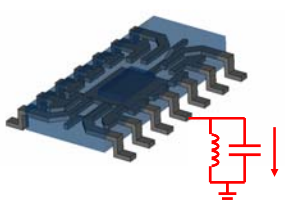
$$\boxed{Slope = \mathrm{Im}\left\{ \frac{\mathbf{w}^T \mathbf{v}}{\mathbf{w}^T \mathbf{M}' \mathbf{v}} \right\}}$$



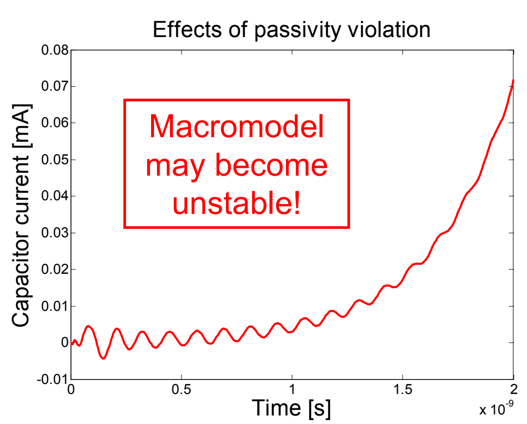
$\mathbf{w},\mathbf{v}$: Left and right eigenvectors of $\mathbf{M}$ associated to ω_0

$\mathbf{M'}$: Another Hamiltonian matrix (computed via $\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D}$)

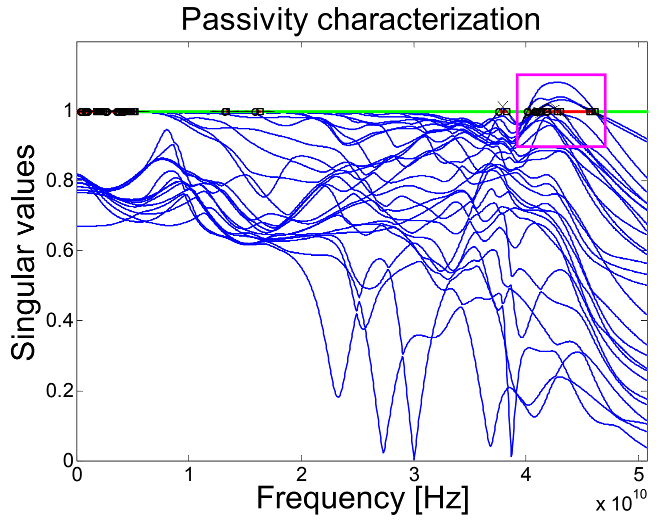
Example 4: change terminations



Port terminations:
R = 50 mΩ ÷ 50 Ω
L = 1 nH
C = 1 pF



Example 4: passivity characterization



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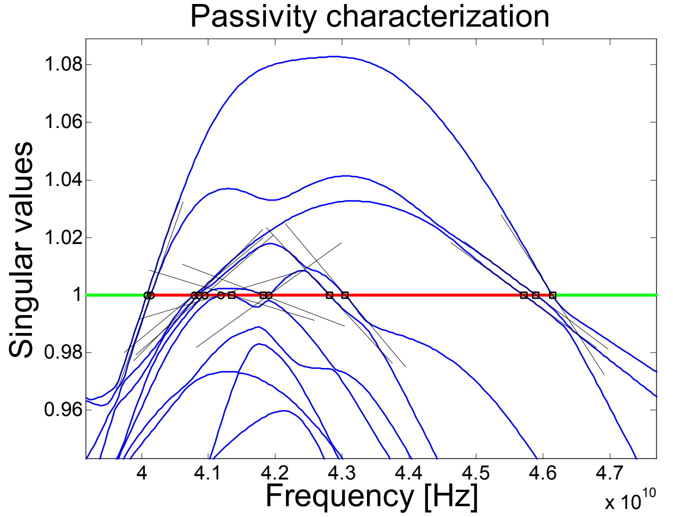
Example 4: passivity characterization



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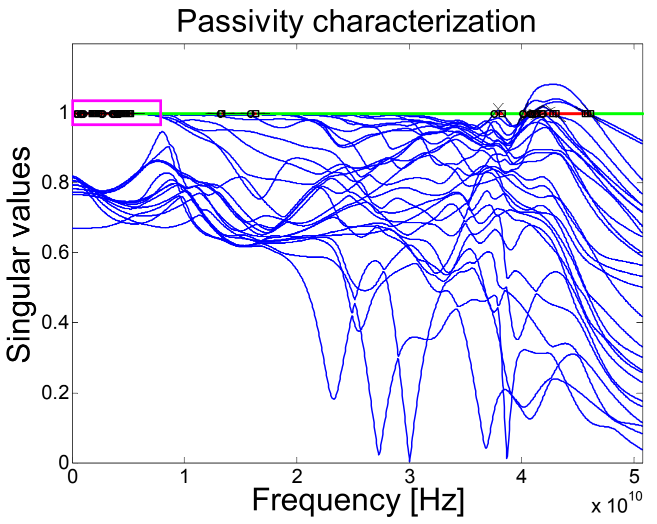
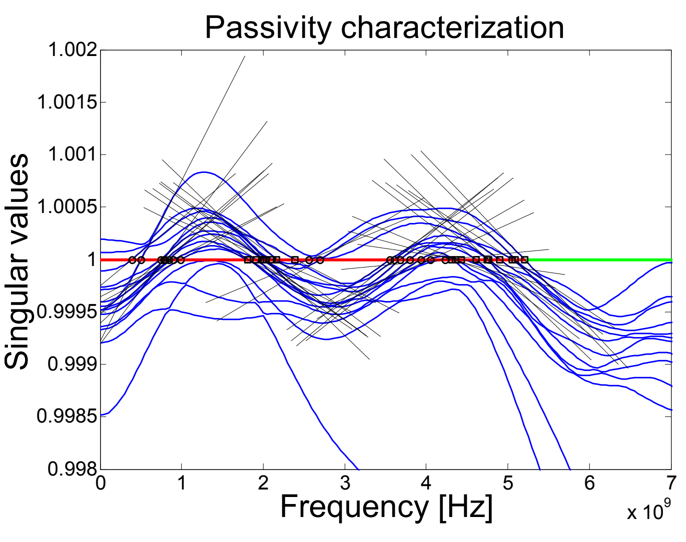
Example 4: passivity characterization



Example 4: passivity characterization



Passivity enforcement

- Generate a new passive macromodel
- Apply small correction to preserve accuracy
  - original dataset should be passive
  - original macromodel should be accurate
  - (usually) preserve poles

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{B}\;\mathbf{x} \\ \mathbf{y} = \mathbf{C}\,\mathbf{w} + \mathbf{D}\;\mathbf{x} \end{cases} \quad \Longrightarrow \quad \begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{B}\;\mathbf{x} \\ \mathbf{y} = (\mathbf{C} + \mathbf{dC})\,\mathbf{w} + \mathbf{D}\;\mathbf{x} \end{cases}$$

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Passivity enforcement

Several different approaches are possible. Examples are

Quadratic/convex optimization

[B.Gustavsen, A.Semlyen: IEEE Trans. Power Systems, vol.16, 2001]
[C.P.Coelho, J.Phillips, L.M.Silveira, IEEE Trans. CADICAS, vol.23, 2004]

Trace parameterization/Semi-Definite Programming

[H.Chen, J.Fang: Proc. EPEP, 2003]

Perturbation of residues

[D.Saraswat, R.Achar, M.Nakhla: Proc. EPEP, 2003]

Perturbation of Hamiltonian eigenvalues

[S.Grivet-Talocia: Proc. SPI, 2003 and IEEE Trans. CAS (in press)]

Many others… Hot research topic!

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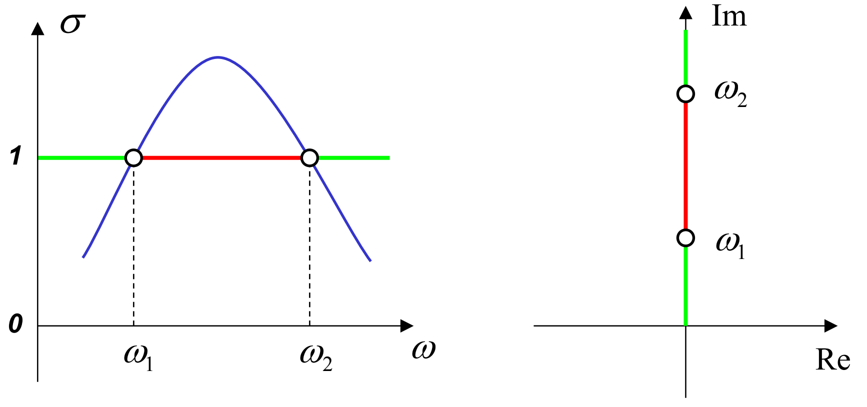
Perturbation of Hamiltonian Eigs

Singular values of H

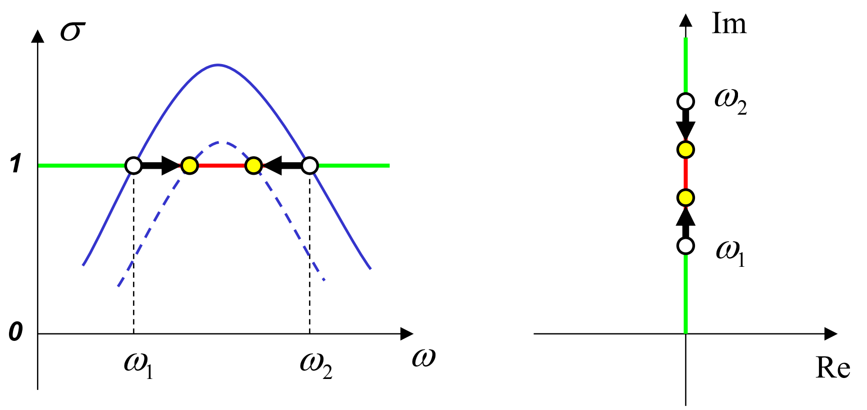
Eigenvalues of M



Perturbation of Hamiltonian Eigs

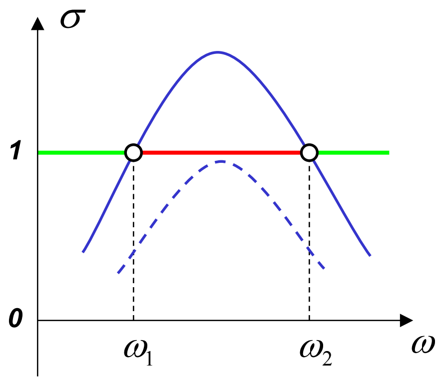
Singular values of H

Eigenvalues of M

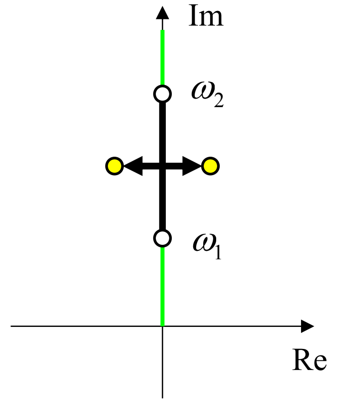


Perturbation of Hamiltonian Eigs

Singular values of H



Eigenvalues of M



Perturbation of Hamiltonian Eigs

First-order perturbation of eigenvalues (again)

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \\ \mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u} \end{cases}$$

Perturb state matrix $\mathbf{C}$

$$\tilde{\mathbf{C}} = \mathbf{C} + \mathbf{dC}$$

$\tilde{\mathbf{M}} \approx \mathbf{M} + \mathbf{dM}$ (first-order: $\mathbf{dM}$ is linear in $\mathbf{dC}$)

$$\mathbf{w}_m^T\,\mathbf{dM}\,\mathbf{v}_m \approx j(\tilde{\omega}_m - \omega_m)\,\mathbf{w}_m^T \mathbf{v}_m$$

Linear constraint on the correction matrix $\mathbf{dC}$

Perturbation of Hamiltonian Eigs

Iterate

$$\begin{cases} \boxed{\mathbf{Z}}\,\boxed{\mathbf{c}} = \boxed{\mathbf{r}} \\ \min\ \mathbf{c}^T \boxed{\mathbf{Q}}\, \mathbf{c} \end{cases}$$

Until passive

Linear coefficients

Entries in matrix $\mathbf{dC}$

Perturbations of eigenvalues

Related to controllability Gramian

Minimizes the perturbation on the original responses

Preserve accuracy of macromodel

Minimize this norm !

$$\sum_{i,j} \int_0^\infty \left(\tilde{h}_{i,j}(t) - h_{i,j}(t)\right)^2 \mathrm{d}t \quad = \quad \| \mathbf{dC}\ \mathbf{W}\ \mathbf{dC}^T \|_F^2$$

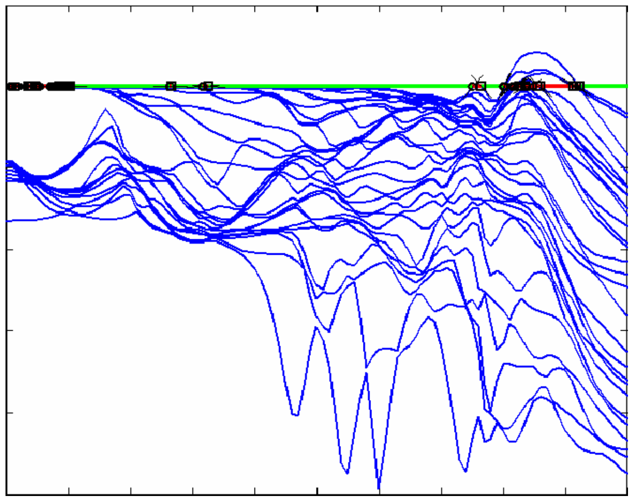
Induced perturbation in the impulse responses

Weighted norm of state matrix perturbation

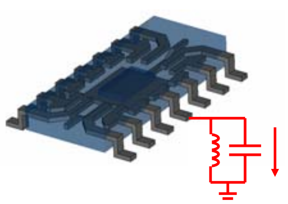
$\mathbf{W}$: controllability Gramian

$$\mathbf{AW} + \mathbf{WA}^T = -\mathbf{BB}^T$$

Example 4: passivity compensation



Example 4 : passivity compensation

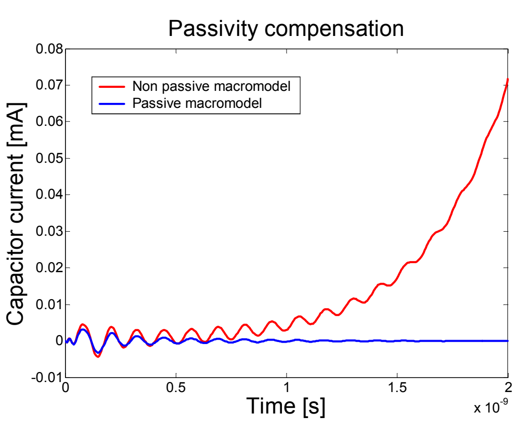


Port terminations:

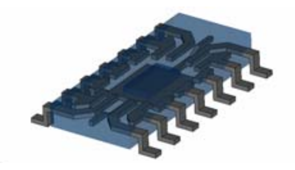
$R = 50\ \mathrm{m}\Omega \div 50\ \Omega$

$L = 1\ \mathrm{nH}$

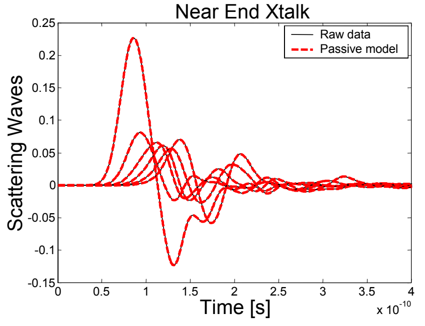
$C = 1\ \mathrm{pF}$

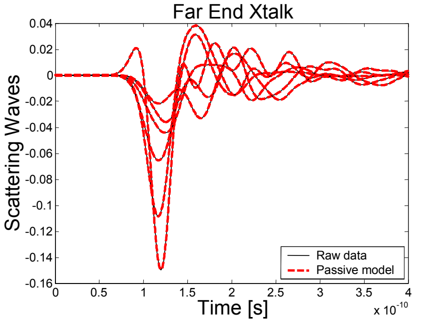


Example 4: passive macromodel



Raw data → Macromodel → Passive macromodel





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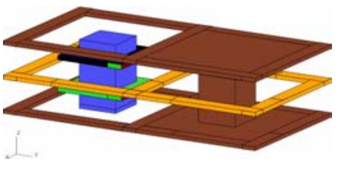
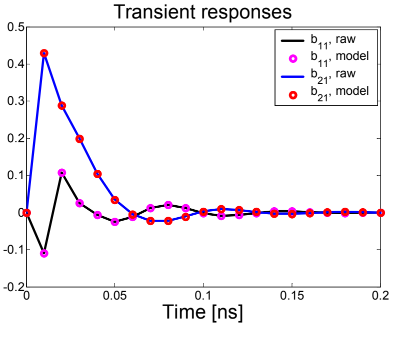
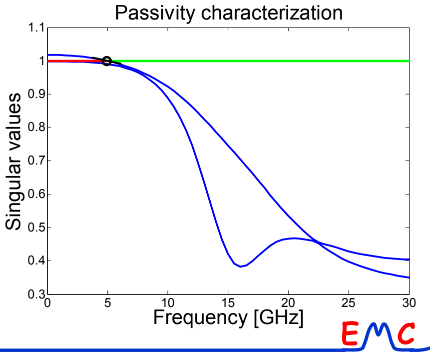
Example 1: single via, nonpassive



Raw data:
Triangle Impulse Responses obtained by a transient PEEC solver (by Dr. Ruehli, IBM)





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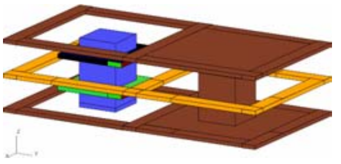
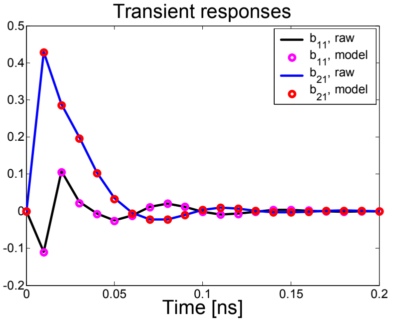
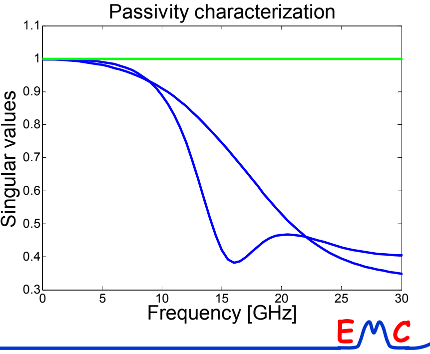
Example 1: single via, passive



Raw data:
Triangle Impulse Responses obtained by a transient PEEC solver (by Dr. Ruehli, IBM)





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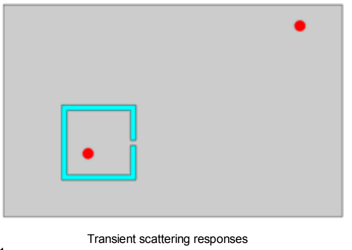
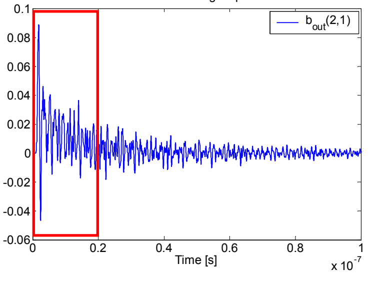
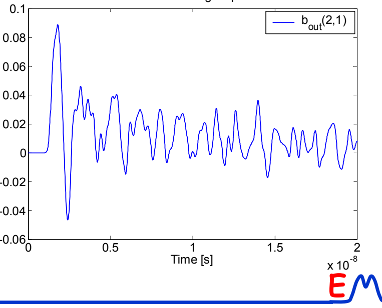
Example 2: segmented power bus



- 2-port structure
- Time-Domain solution
- CST Microwave Studio
- Bandwidth: 3 GHz
- 50 Ω port terminations





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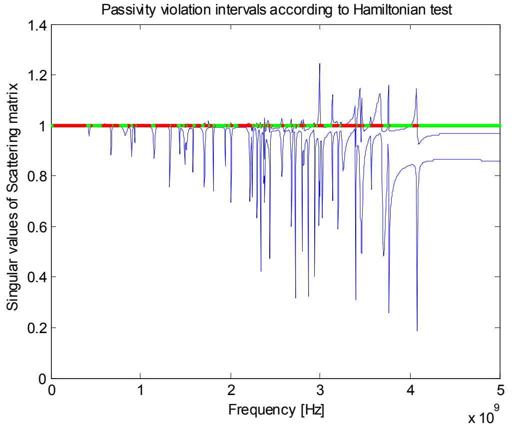
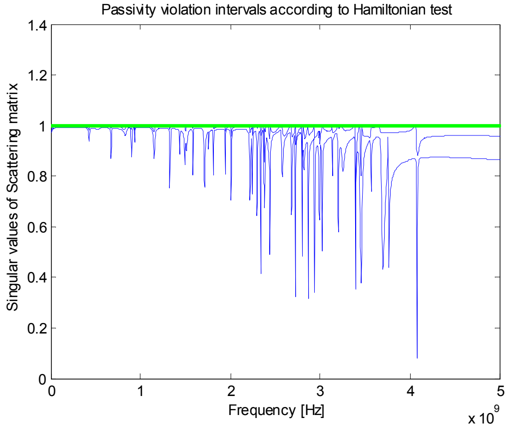
Example 2: segmented power bus

Passivity characterization



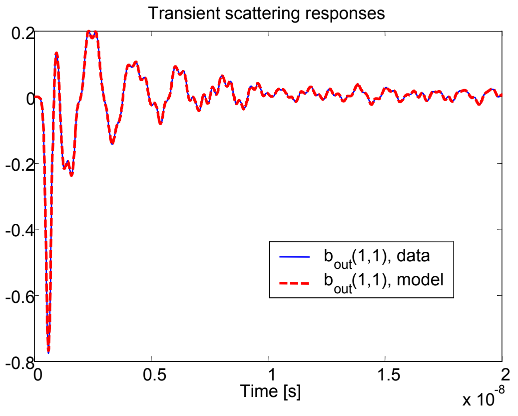
Example 2: segmented power bus

Passivity compensation



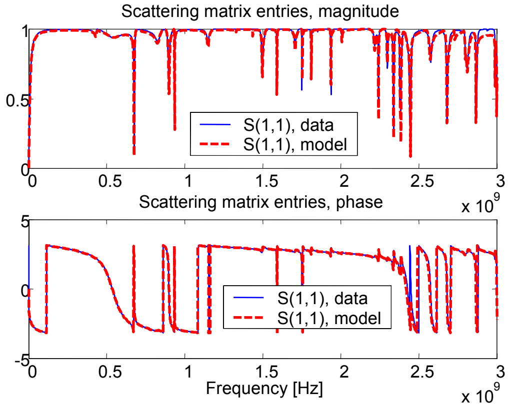
Example 2: segmented power bus

80-poles **passive** model



Example 2: segmented power bus

Comparison vs. frequency-domain scattering data



More examples…

41 poles, 2 ports

Compensation | Accuracy

110 poles, 5 ports

Compensation | Accuracy

308 poles, 11 ports

Compensation | Accuracy

Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

Macromodel implementation

Main approaches

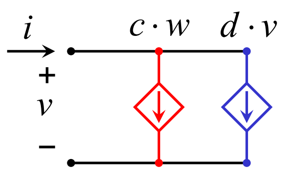
1. Synthesize an equivalent circuit in SPICE format
   - No access to SPICE kernel
   - Must use standard circuit elements
2. Direct SPICE implementation via recursive convolution
   - Laplace element, most efficient
3. Other languages for mixed-signal analyses
   - Verilog-AMS, VHDL-AMS, …
   - Equation-based

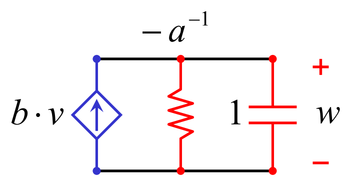
SPICE synthesis

Admittance representation

One-port, one-pole

$$\begin{cases} \dot{w} = a\,w + b\,v \\ i = c\,w + d\,v \end{cases}$$





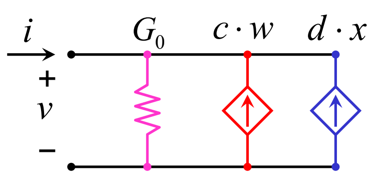
SPICE synthesis

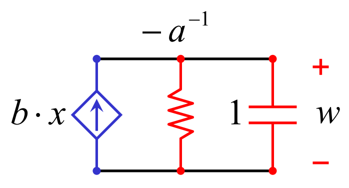
Scattering representation

One-port, one-pole

$$\begin{cases} \dot{w} = a\,w + b\,x \\ y = c\,w + d\,x \end{cases}$$

$$x = G_0 v + i,\; y = G_0 v - i$$





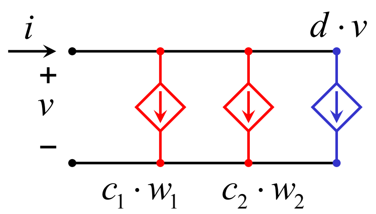
SPICE synthesis

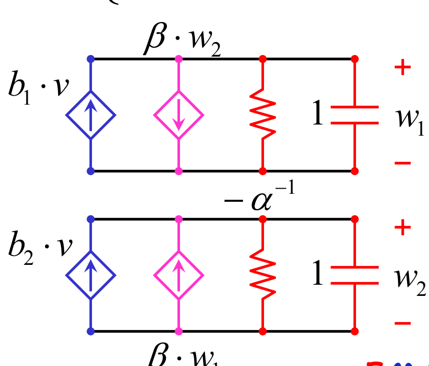
Admittance representation

One-port, two-poles (complex)

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{b}\,v \\ i = \mathbf{c}\,\mathbf{w} + d\,v \end{cases}$$

$$p_{1,2} = \alpha \pm j\beta$$



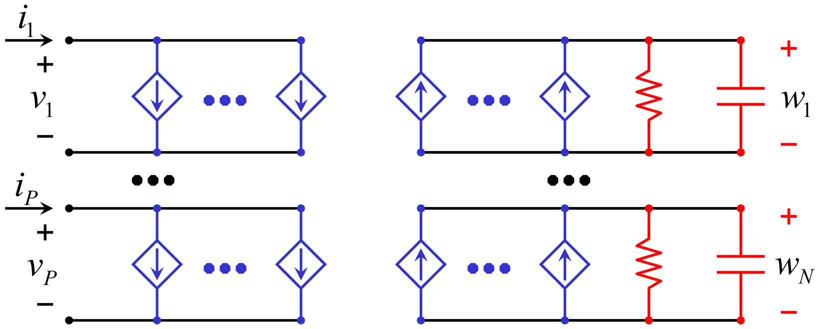


SPICE synthesis

Admittance representation

General state-space synthesis

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A}\,\mathbf{w} + \mathbf{B}\,\mathbf{v} \\ \mathbf{i} = \mathbf{C}\,\mathbf{w} + \mathbf{D}\,\mathbf{v} \end{cases}$$



Recursive convolutions

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} = \mathbf{D} + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

$$\mathbf{h}(t) = \mathbf{D}\delta(t) + \sum_n \mathbf{R}_n e^{p_n t} u(t)$$

$$\mathbf{y}(t) = \mathbf{D}\mathbf{x}(t) + \sum_n \mathbf{R}_n \int_0^t e^{p_n(t-\tau)}\mathbf{x}(\tau)\,\mathrm{d}\tau$$

Recursive convolutions

$$\tilde{\mathbf{y}}(t_k) = \int_0^{t_k} e^{p(t_k-\tau)}\mathbf{x}(\tau)\mathrm{d}\tau$$

Discrete time
$t_k = t_{k-1} + \Delta t_k$

$$= \int_0^{t_{k-1}} e^{p(t_k-\tau)}\mathbf{x}(\tau)\mathrm{d}\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k-\tau)}\mathbf{x}(\tau)\mathrm{d}\tau$$

$$= e^{p\Delta t_k}\int_0^{t_{k-1}} e^{p(t_{k-1}-\tau)}\mathbf{x}(\tau)\mathrm{d}\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k-\tau)}\mathbf{x}(\tau)\mathrm{d}\tau$$

Approximation!

$$\approx e^{p\Delta t_k}\tilde{\mathbf{y}}(t_{k-1}) + \frac{1-e^{p\Delta t_k}}{p}\mathbf{x}(t_k)$$

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The macromodeling dream…

Arbitrary characterization of the structure

- Equation-based or Black-Box
- Time or frequency, simulation or measurement

Generation of a broadband macromodel

- Any order, any number of ports
- Any prescribed accuracy
- Stable and passive by construction
- Efficient (reduced-order and low-complexity)
- Fully automatic

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