Nonadiabatic geometrical quantum gates in semiconductor quantum dots

Nonadiabatic geometrical quantum gates in semiconductor quantum dots

Paolo Solinas,[1] Paolo Zanardi,[2] Nino Zanghì,[1] and Fausto Rossi[2,3]
[1]*Istituto Nazionale di Fisica Nucleare (INFN) and Dipartimento di Fisica, Università di Genova, Via Dodecaneso 33, 16146 Genova, Italy*
[2]*Institute for Scientific Interchange (ISI), Viale Settimio Severo 65, 10133 Torino, Italy*
[3]*Istituto Nazionale per la Fisica della Materia (INFM) and Dipartimento di Fisica, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*

(Received 20 January 2003; published 23 May 2003)

In this paper, we study the implementation of nonadiabatic geometrical quantum gates with in semiconductor quantum dots. Different quantum information enconding (manipulation) schemes exploiting excitonic degrees of freedom are discussed. By means of the Aharanov-Anandan geometrical phase, one can avoid the limitations of adiabatic schemes relying on adiabatic Berry phase; fast geometrical quantum gates can be, in principle, implemented.

DOI: 10.1103/PhysRevA.67.052309 PACS number(s): 03.67.Lx

I. INTRODUCTION

The holonomic quantum computation proposal (HQC) [1] recently led to a number of investigations [2] aimed to assess its feasibility. At variance with "ordinary" dynamical quantum gates, the Holonomic ones depend only on geometrical features (i.e., the angle swept by a vector on a sphere) of a suitable quantum control process. It has been argued that HQC might lead to computational schemes more robust against some class of errors. Despite the fact that this crucial property has not been clearly demonstrated so far (for a critical view, see e.g., Ref. [3]), HQC surely provides a sort of an intermediate step towards topological quantum computing [4,5]. The latter represents an intriguing and ambitious paradigm for inherently fault-tolerant QC.

Many proposals for practical HQC follow the adiabatic approach [2]; it consists in changing the Hamiltonian parameters in order to produce a loop in the Hamiltonian space $[H(0)=H(T)]$. For an adiabatic evolution, if we start from an eigenstate $|n(0)\rangle$ of $H(0)$ with eigenvalue $E_n(0)$, during the evolution we remain in the instantaneous eigenvector $|n(t)\rangle$ of $H(t)$ with eigenvalue $E_n(t)$. At the end of the loop, the state will differ by the initial state only for a phase factor (Berry phase). If the eigenstate is degenerate, we end up in a superposition of the degenerate states and then we have a non-Abelian holonomic operator [6].

On the other hand, it is well known that the major obstacle against the practical realization of quantum information processing (QIP) [7] is provided by the detrimental interaction with environmental degrees of freedom. This interaction results, typically in an extremely short time, in the destruction of the quantum coherence of the information-encoding quantum state, which in turns spoils the computation [7]. It follows that for QIP purposes, it is very important to have fast logical gates to be able to realize numerous logical operations within decoherence time.

The fact that we have to change parameters slowly is an obvious drawback of the adiabatic approach. Then, the possibility of having geometrical gate without the adiabatic limitation looks very appealing.

In 1987 Aharanov and Anandan (A-A) [8] showed that there is an additional geometrical phase factor for *all* the cyclic evolution of the states (not only for the adiabatic ones). The A-A phase is a generalization of the Berry phase, and we recover this when the adiabatic condition is restored. Recently, some proposals for nonadiabatic geometrical gates have been made [9].

In this paper, we shall propose a universal set of nonadiabatic geometrical gates using excitonic states in semiconductor quantum dots. The schemes illustrated below rely on the physical setup analyzed in Ref. [10] and on the abstract geometrical structure of Ref. [11].

II. EXCITON–NO EXCITON QUBIT

In Ref. [12], it has been shown how excitonic states in a quantum dot can be used to perform universal QIP. The logical states were the ground state $|G\rangle$ and the excitonic state $|E\rangle$, and they were driven by *all-optical* control (with ultrafast laser). Even if the decoherence time in this system is quite short, the ultrafast laser technology used for the coherent manipulations allows, in principle, to perform a large number of operations.

Let us start by showing how the scheme by Qi *et al.* [11] can be applied in this semiconductor context. We have a two-level system ($\hbar=1$ and ω_0 energy separation) interacting with a laser field (radiation-matter interaction) and, then, the interaction Hamiltonian can be written as

$$H_{int}=-[\Omega e^{-i\omega_L t-\phi}|E\rangle\langle G|+\text{H.c.}]. \tag{1}$$

In a rotating frame (with precession frequency ω_L), the total Hamiltonian is (using "spin" formalism) $H_R=\mathbf{B}\cdot\boldsymbol{\sigma}$, with $\mathbf{B}=[\Omega\cos\phi,\Omega\sin\phi,(\omega_0-\omega_L)/2]$ and $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$. This is the Hamiltonian presented in Ref. [11] and, then, we can obtain the same gates. With $\mathbf{B}\neq 0$ the spin will precede on the Bloch sphere on a plane orthogonal to $\mathbf{B}$ according the Bloch's equations.

Following Ref. [11], it is easy to see that—by choosing the laser parameters (phase and frequencies) in a suitable way—one can produce a sequence of laser pulses that enact a loop on the Bloch sphere; the final state will acquire a geometrical phase independent of the velocity during the traversed loop (no adiabatic approximation). The final operator depends on the angle swept on the sphere by the state vector during the evolution. With a sequence of two π-pulses, we can obtain two single-qubit gates. First, we take $\omega_L \neq \omega_0$ (off-resonant laser) and then produce two π-pulses with different phases (i.e., $\Delta\phi = \pi$) and obtain the following gate :

$$|0\rangle \rightarrow \cos\gamma|0\rangle - \sin\gamma|1\rangle,$$

$$|1\rangle \rightarrow \cos\gamma|1\rangle + \sin\gamma|0\rangle, \tag{2}$$

where γ is half the angle swept by the vector on the Bloch sphere and it depends on the gate parameters (i.e., the laser frequency) $\gamma = 2\arctan(2\Omega/(\omega_0 - \omega_L))$.

For a *selective phase* gate, we have a resonant condition $\omega_0 = \omega_L$ and produce two π-pulses with opposite phases $(\phi_1 = -\phi_2 = \phi_0)$, and we have

$$|0\rangle \rightarrow e^{i\tilde{\gamma}}|0\rangle,$$

$$|1\rangle \rightarrow e^{-i\tilde{\gamma}}|1\rangle, \tag{3}$$

where $\tilde{\gamma} = 2\phi_0$.

We note that the dynamical phase factor in standard geometric quantum computation must be eliminated with several adiabatic loops in order to let the phase factor cancels each other. In this model, it does not appear because the motion on the Bloch sphere is on a plane orthogonal to **B**, and so it can be easily shown that $\langle\psi|H|\psi\rangle = 0$ and the dynamical phase factor is zero. Of course, these geometric gates are much faster than the adiabatic ones [10] that had the limitation of the slow change of parameters.

This kind of geometrical manipulation of excitonic-encoded information should be easier to implement and to verify experimentally, because they are just produced by a sequence of π-pulses with constant parameters (frequency or phase of the laser) with just one laser instead of three lasers in which the intensity and the phase change during the evolution.

For the two-qubit gate, we have to exploit qubit-qubit interaction in order to construct nontrivial operators; then every system has different implementation of such gates. Since we work with semiconductor excitons, we use exciton-exciton dipole interaction.

Let us consider two dots with exciton energy $\omega_0/2$ (the energy is rescaled in order to have $-\omega_0/2$ for the ground states). If the two dots are coupled, the presence of an exciton in one of them causes an energetic shift δ in the other because of the dipole-dipole interaction. States with a single exciton are not shifted. The energy levels are shown in Fig. 1. The Hamiltonian accounting for the biexcitonic shift is $H_0 = (\omega_0 + \delta)|EE\rangle\langle EE| - \omega_0|GG\rangle\langle GG|$.

The dipole interaction between dots can be used to construct nontrivial two-qubit gates both dynamical [12] and geometrical [10]. In fact, if we use two lasers tuned to the two-exciton state transition $[\omega_L^1 = \omega_L^2 = (\omega_0 + \delta)/2]$, we can avoid single-photon processes (which produce $|EG\rangle$ and $|GE\rangle$ states) and favor only two-photon processes (which produce $|EE\rangle$).

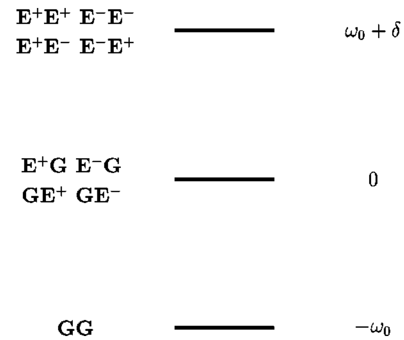

FIG. 1. Energy levels for two coupled dots with dipole-dipole interaction. δ is the biexcitonic shift.

The effective interaction Hamiltonian for the two-photon process is

$$H_{int} = -\frac{2\hbar^2}{\delta}\Omega_+^2 e^{-i(\omega_{L,1}+\omega_{L,2})} e^{-i(\phi_1+\phi_2)}|E\rangle\langle G|^{\otimes 2} + \text{H.c.}, \tag{4}$$

where $\omega_{L,i}$ and ϕ_i are the frequency and the phase of the laser *i*.

The total Hamiltonian is similar to that in Eq. (1) and, then, using a properly chosen sequence of synchronous pulses [so that the two-photon Rabi frequencies in Eq. (4) simulate the one in Eq. (1)], we can apply a phase gate similar to that in Eq. (3) and complete the universal set of quantum gates.

III. EXCITON SPIN QUBIT

A further excitonic encoding can be obtained following the spin-based scheme presented in Ref. [10]. There, a four-level system with three degenerate excited states ($|E^{\pm}\rangle$ and $|E^0\rangle$) and a ground state ($|G\rangle$) was used; the excitonic states were connected with $|G\rangle$ by three different lasers with circular ($\pm$) and linear (along z axis) polarization and, modulating the phase and the frequency of the three lasers, we were able to construct adiabatic holonomic gates.

To obtain nonadiabatic geometrical gates, in this system, the basic idea is to encode logical information in two degenerate exciton states with different total angular momenta, i.e., $|E^{\pm}\rangle$. The extension of the previous gating model is not completely straightforward; in fact, the logical qubits $|E^+\rangle$ and $|E^-\rangle$, due to angular-momentum conservation in radiation-matter interaction, are not directly, i.e., by a one-photon ladder operators, connected.

In order to circumvent this problem and to enact such a ladder operator, one can resort to an off-resonant two-photon Raman process. This is a standard trick in quantum optics. Each quantum dot is shined by a couple of lasers having polarizations $+$ and $-$, and a frequency with a detuning Δ with respect to the excitonic transition energy. The level scheme with the associate transition is shown in Fig. 2. Provided that $\Omega_{\pm} \ll \Delta$ (the $\Omega_{\pm}$'s are the laser Rabi frequencies), first-order processes are then strongly suppressed; the dynamics is well described by the following second-order effective Hamiltonian

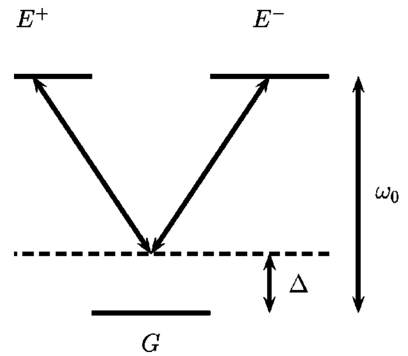

FIG. 2. Connection of the logical subspaces E^+ and E^-. The Δ is the detuning of the lasers that allows us to connect the two states through the Raman transition.

$$H_{eff}=\frac{\Omega_+\Omega_-}{\Delta}|E^+\rangle\langle E^-|+\text{H.c.} \tag{5}$$

It should be now clear—since the above Hamiltonian structure is same as that of Eq. (1)—that even for this kind of excitonic encoding using different polarizations, one can realize all the required single-qubit operations.

Another single-qubit gate that can be implemented easily is the *phase-shift* gate. Our scheme has *a priori* separated subspaces because of the different response to polarized laser. So, if we want $|E^+\rangle$ to get a phase factor, we can just switch the $+$ laser to resonant frequency, and then apply a pulse sequence that produces gate 2. Since we can neglect the phase accumulated by $|G\rangle$ and no phase is accumulated by $|E^-\rangle$, the gate operator will be $U=\exp(i\tilde{\gamma}|E^+\rangle\langle E^+|)$, where, as before, $\tilde{\gamma}$ is half the solid angle swept in the evolution. These two gates complete the single-qubit gate set.

Finally, to obtain a universal set of quantum logical gates, we must construct a two-qubit gates. The easiest to be implemented in our model is a *selective phase gate*. As shown before, using lasers resonant with the two exciton with positive polarization, we can select two-photon processes and couple only the $|E^+E^+\rangle-|GG\rangle$ states [10]. The effective Hamiltonian for these two-photon processes is similar to that in Eq. (4) with $|E^+\rangle$ instead of a generic exciton state $|E\rangle$.

The two lasers are polarized with $+$ polarization and follow the pulse sequence for gate 1; the final geometric operator will be $U=\exp(i\tilde{\gamma}|E^+E^+\rangle\langle E^+E^+|)$, where $\tilde{\gamma}$ is half the angle swept on the Bloch sphere in the $|E^+E^+\rangle-|GG\rangle$ space.

A few remarks are now in order regarding the different kind of excitonic polarization we have considered so far. In the second—polarization-based—encoding we need more laser pulses (and then longer time for the application of the gates) with respect to the model following the first scheme with nonpolarized excitons. This makes the setup slightly more complicated, but now the logical 1 and 0 states correspond here to energetically degenerate states with the same orbital wave function structure. This fact should (1) make the qubit more robust against *pure* dephasing processes (2) set to zero the qubit self-Hamiltonian, i.e., the σ_z component allowing for a simplified gate design and, then, no recoupling pulse are required.

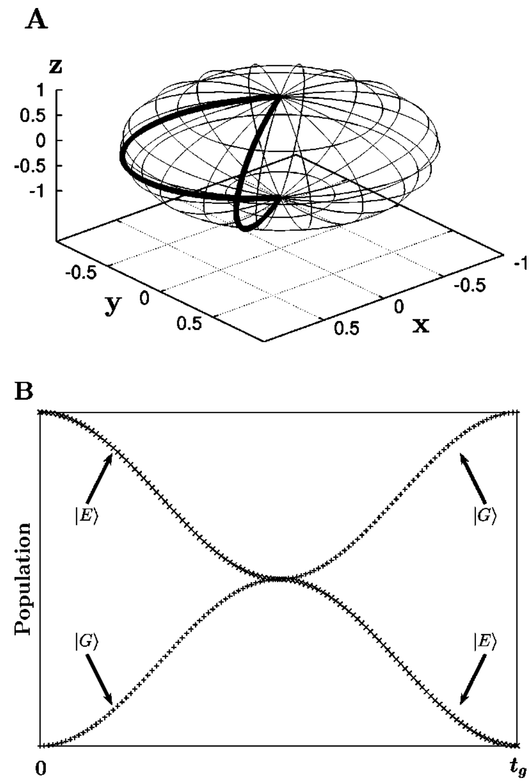

FIG. 3. Gate 1 for the unpolarized excitons model. The parameters are chosen in order to obtain a NOT gate. (a) Evolution of $|E\rangle$ state on the Bloch sphere. (b) Population evolution for the logical states $|E\rangle$ and $|G\rangle$.

On the other hand, it should be noted that in the second scheme, both the code words correspond to *unstable* states, indeed excitons will eventually recombine through the semiconductor gap by emitting a photon. On the contrary, in the first encoding scheme, the logical 0 corresponds to the ground state $|G\rangle$ of the crystal, and it is therefore a stable state.

Exciton recombination corresponds in the first scheme to the amplitude-damping process $|1\rangle\mapsto|0\rangle$. One can take care of this kind of environment-induced error by both the techniques of quantum error correction [13] or error avoiding [14] depending on the spatial symmetry of the damping process. Using polarization encoding, spontaneous decay gives rise to a leakage to the computational subspace in the ground state of the crystal $|G\rangle$ is no longer a computational code word. In this case, one can resort to leakage-elimination strategies based on active intervention on the system [15].

IV. SIMULATIONS

To test our models, we performed numerical simulations of the quantum gates solving the Schrödinger equation. For the first model (with no polarized excitons), we took $|E\rangle$ as starting state and then simulated the evolution when we applied the pulse sequences presented. In Fig. 3, the results of the simulation for gate 1 are shown; the parameters are cho-

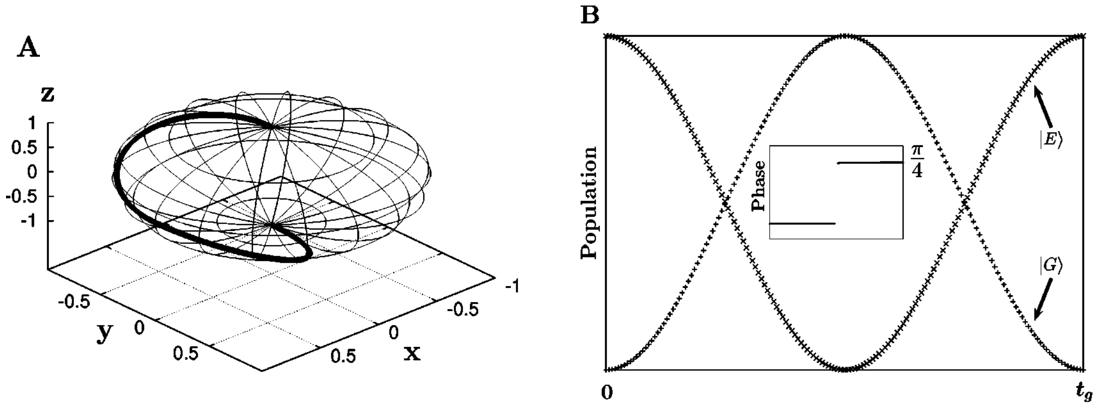

FIG. 4. Gate 2 for the unpolarized excitons model. (a) Evolution of $|E\rangle$ state on the Bloch sphere. (b) Population evolution for the logical states $|E\rangle$ and $|G\rangle$.

sen in order to obtain a NOT gate. In Fig. 3(a), the curve traversed by the state in the Bloch space and in Fig. 3(b) the population evolutions are presented. Once decided which gate has to apply, we can have an estimate of the gate time. For this NOT gate, the laser frequency is not resonant and is constrained by the gate choice ($\omega_L = \omega_0 - 2\Omega$); the time gate is fixed by the Rabi frequency of the laser. For realistic laser parameters ($\Omega^{-1} = 50$ fs), we have $t_{gate1} = 0.1$ ps.

In Fig. 4, we show (for gate 2) the loop in the Bloch space; the population evolutions in Fig. 4(a) and the phase accumulated during the evolution (inset) Fig. 4(b). The parameters are chosen in order to obtain $\tilde{\gamma} = \pi/4$, and the final state is $(1+i)/\sqrt{2}|E\rangle$. The laser frequency is resonant with the transition ($\omega_L = \omega_0$), and with the same Rabi frequency used before we have $t_{gate2} = 0.15$ ps.

In the second model, first we have to test the validity of the approximation used in Eq. (5); for this purpose, we simulated the evolution of the three-level system showed in Fig. 2 and show the result in Fig. 5. We choose $\Delta/\Omega = 10$ ($\Omega_+ = \Omega_- = \Omega$) and, as we can see, this is sufficient to avoid population of $|G\rangle$ state and to have the standard Rabi oscillations between the logical states.

We note that, because of the perturbative request in Eq. (5), the *effective* magnetic field **B** has small x and y components, and then a sequence of two π-pulses is not sufficient to construct a generic superposition of logical qubits. Even if the geometrical phase accumulated during the loop is small, it is sufficient to iterate the procedure to apply the desired geometrical operator. Using the same perturbation parameter as in Eq. (5), we simulate the evolution of $|E^+\rangle$. In Fig. 6, we show the population evolutions of the states $|E^+\rangle - |E^-\rangle$ when they are subjected to a π-pulse sequence in order to obtain a NOT gate. Of course the gating time in this situation depends on which gate we want to apply and the parameter used in the model.

V. CONCLUSIONS

In summary, we proposed two approaches to geometric nonadiabatic quantum information processing in semiconductor quantum dots. In both the cases, we have been able to construct a universal set of quantum gates using the Aharonov-Anandan phase. In the first scheme, the qubit is realized by the presence or absence of a (ground) state exciton. A coupling with an external laser field allows for the nonadiabatic realization of the geometrical gates. The dipole-dipole coupling between excitons plays an essential role in action of the entangling two-qubit gate.

In the second approach, we encode information in degenerate states using, as quantum degree of freedom, the polar-

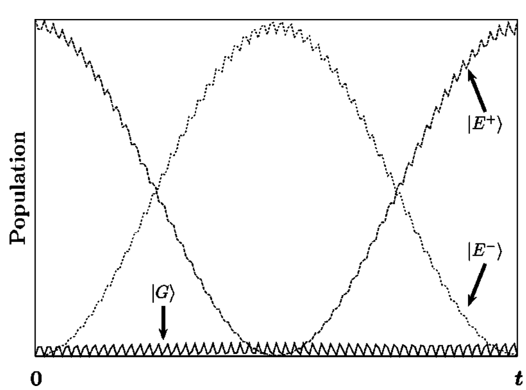

FIG. 5. Population evolutions for the three-level system with polarized (logical) excitons $|E^+\rangle$, $|E^-\rangle$, and $|G\rangle$ with lasers with a Δ detuning. The perturbative parameter is $\Delta/\Omega = 10$.

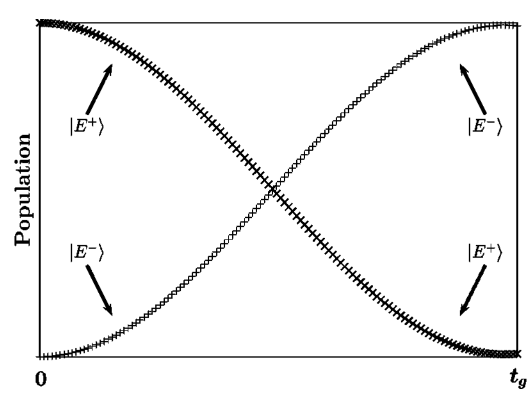

FIG. 6. Populations of logical states for polarized exciton model. The phase accumulated in a single loop is $\gamma = 0.027\,025\,4$ and we iterate the cycle of π-pulse 59 times to obtain a NOT gate.

ization, i.e., total spin, of the excitons ($|E^{\pm}\rangle$). The logical states are not directly connected, but we showed first how to avoid this problem with two-photon (Raman) transition and second how to implement in this way a *selective phase* gates (for one and two qubits). Numerical simulations with realistic parameters show that these gates can be, in principle, enacted within the decoherence time. The models for nonadiabatic (fast) QIP presented in this paper combine the features of geometrical gates with the ultrafast gate control possible in semiconductor nanostructures; an experimental verification of these schemes seems under the reach of current technology.

[1] P. Zanardi and M. Rasetti, Phys. Lett. A **264**, 94 (1999); J. Pachos, P. Zanardi, and M. Rasetti, Phys. Rev. A **61**, 010305(R) (2000).

[2] J. Pachos and S. Chountasis, Phys. Rev. A **62**, 052318 (2000); L.-M. Duan, J.I. Cirac, and P. Zoller, Science (Washington, DC, U.S.) **292**, 1695 (2001); D. Ellinas and J. Pachos, Phys. Rev. A **64**, 022310 (2001); I. Fuentes-Guridi *et al.*, *ibid.* **66**, 022102 (2002); A. Recati *et al.*, *ibid.* **66**, 032309 (2002); J. Pachos, *ibid.* **66**, 042318 (2002); L. Faoro, J. Siewert, and R. Fazio, e-print cond-mat/0202217.

[3] A. Nazir, T.P. Spiller, and W.J. Munro, Phys. Rev. A **65**, 042303 (2002); A. Blais, A.-M.S. Tremblay, e-print quant-ph/0105006.

[4] A. Kitaev, e-print quant-ph/9707021; M.H. Freedman, A. Kitaev, and W. Zhenghan, Commun. Math. Phys. **227**, 587 (2002); Michael H. Freedman, Michael Larsen, and Zhenghan Wang, *ibid.* **227**, 3 (2002).

[5] P. Zanardi and S. Lloyd, e-print quant-ph/0208132; Phys. Rev. Lett. (to be published).

[6] F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984).

[7] For reviews, see D.P. DiVincenzo and C. Bennet, Nature (London) **404**, 247 (2000); A. Steane, Rep. Prog. Phys. **61**, 117 (1998).

[8] Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987).

[9] X.B. Wang and M. Keiji, Phys. Rev. B **65**, 172508 (2002); J. Phys. A **34**, L631 (2001); Phys. Rev. Lett. **87**, 097901 (2001); S.-L. Zhu and Z.D. Wang, *ibid.* **89**, 097902 (2002); Phys. Rev. A **66**, 042322 (2002).

[10] P. Solinas *et al.*, Phys. Rev. B **67**, 121307(R) (2003).

[11] X.-Q. Li *et al.*, Phys. Rev. A **66**, 042320 (2002).

[12] E. Biolatti, R.C. Iotti, P. Zanardi, and F. Rossi, Phys. Rev. Lett. **85**, 5647 (2000); E. Biolatti, I. D'Amico, P. Zanardi, and F. Rossi, Phys. Rev. B **65**, 075306 (2002).

[13] E. Knill and E. Laflamme, Phys. Rev. A **55**, 900 (1997); D. Gottesman, *ibid.* **54**, 1862 (1996); A.R. Calderbank *et al.*, Phys. Rev. Lett. **78**, 405 (1997).

[14] L.M. Duan and G.C. Guo, Phys. Rev. Lett. **79**, 1953 (1997); P. Zanardi and M. Rasetti, *ibid.* **79**, 3306 (1997); D.A. Lidar, I.L. Chuang, and K.B. Whaley, *ibid.* **81**, 2594 (1998).

[15] P. Zanardi, Phys. Lett. A **258**, 77 (1999); L. Tian and S. Lloyd, Phys. Rev. A **62**, 050301 (2000); L.-A. Wu, M. Byrd, and D. Lidar, Phys. Rev. Lett. **89**, 127901 (2002).