Learning Curves and p-charts for a preliminary estimation of asymptotic performances of a manufacturing process

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ABSTRACT *This paper presents a method for a preliminary estimation of asymptotic performances of a manufacturing process based on the knowledge of its learning curve estimated during the setting up of* p*-chart. The main novelties of the method are the possibility of estimating the asymptotic variability of a process and providing a simple approach for evaluating the period of revision of process control limits. An application of the method to a real example taken from the literature is also provided.*

Introduction

As a first approximation, a manufacturing process can be described as a framework able to convert input raw material into finished or partly finished products. Mechanisms of transformation, specific for each context, are ruled by a sequence of organized activities that involve interaction among operators, machinery and production equipment. The combined effect of these elements together with the inner and outer influence quantity are the causes of variability in a manufacturing process.

Control charts are a proven technique to provide diagnostic information and to monitor the variability of a process over time. They are used according to two steps: setting up a chart and monitoring the manufacturing process (Duncan, 1986; Montgomery, 1996). The setting up phase requires the detection of 'assignable' causes, the estimation of the natural tolerance and the process control limits.

Usually, the accepted hypothesis is that without assignable causes the process maintains its performance characteristics over time. Assignable causes may determine an average shifting or a change of the process dispersion. Assignable causes can be divided into two categories: 'positive' and 'negative'. We define as positive those which generate an increase of process variability; and negative the causes that operate in the opposite direction ('favourable assignable cause' (Duncan, 1974). Wear and tear phenomena are typical examples of positive assignable causes, while trends or shifts can appear as positive or negative causes.

Each cause has a proper dynamic. However, the process manager observes a global

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combined effect, having no possibility of discriminating a single contribution. During the life of a process we assist with a continuous ‘overlap’ of the two types of causes. The prevalence of positive or negative causes is detectable by means of statistical control charts.

Referring to a generic process, after removing initial out-of-control causes, we usually observe a gradual reduction of variability over time due to the ‘learning’ mechanism. The ‘physiological’ variability shown by a process in the early life period is not the same as that manifested after a learning period on the field: the so-called ‘asymptotic’ variability. The phenomenon occurs since the operators’ *knowledge* about the process flow, the production equipment and the materials becomes more thorough over time, allowing a more efficient allocation of production factors. It is not true, in general, that the asymptotic variability is zero. The variability reduction depends upon the adaptability of the entire organization to changing conditions of the process (Cherrington *et al.*, 1987; Dada & Marcellus, 1994; Fine, 1986; Franceschini & Rossetto, 1995, 1998; Li & Rajagopalan, 1998). However, the mechanism has not the same intensity over time. After a preliminary phase characterized by relatively high learning, we assist with a progressive attenuation. As Box and Luceño say: “It is practically certain that, given appropriate training and empowerment, quality teams can discover better ways to do things” (Box & Luceño, 1997, p. 19).

The asymptotic variability cannot be reduced further since the process is in the condition of maximum ‘efficiency’. Variability reduction is the main factor that pushes the process manager to revise, after a certain period, the process control limits.

Referring, for example, to a generic manufacturing process, what is the asymptotic fraction nonconforming that the process will produce in the best conditions? The problem is particularly important since the knowledge of asymptotic performances can help to address better process resources. From these results, for example, we could decide to redesign or strengthen some specific subsystems or parts of a process.

The paper presents a method for a preliminary estimation of asymptotic performances of a manufacturing process based on the knowledge of its learning curve and information collected during a p-chart setting up. Practical results obtained by the method are finally shown on a real example taken from the literature.

The method

Usually, the implementation of a control chart follows two steps:

- *Phase 1:* control chart setting up;
- *Phase 2:* control limits verification after a preliminary trial stage and process monitoring.

The revision of control limits becomes necessary whenever there are margins to improve process performances. The time period for a revision of process control limits is not *a priori* fixed; it is usually decided on the trend of the process over time (Montgomery, 1996). The ‘photographs’ of a process provided by the two phases of chart implementation may be used to give a preliminary estimation of asymptotic performances of a manufacturing process.

With reference, for example, to a generic process managed by a p-chart, we propose a method for estimating the ‘asymptotic fraction nonconforming’, and the time required to achieve it.

The general assumption is that the learning mechanism, which can determine a process improvement, follows some evolutionary laws that are not dependent on the specific application context. Learning curves provide a means to observe and track that improvement (Abernathy & Wayne, 1974; Adler & Clark, 1991; Kantor & Zangwill, 1991; Mukherjee *et al.*, 1998).

A literature survey shows a wide variety of studies about particular aspects of learning curves: the effect of prior experience (Lippert, 1976; Cherrington *et al.*, 1987), the relearning mechanism (Bailey & McIntyre, 1997), the setting of performance standards for productivity improvements achieved during the learning stage (Cherrington *et al.*, 1987) and so on.

The concept of the learning curve has been used extensively by economists, management scientists and engineers in analysing production processes. The main areas of investigation have been the empirical measurement of learning curve, the economic implication of this phenomenon and its use in improving managerial decisions. Detailed surveys can be found in Venezia (1985) and Muth (1986).

The most common models of learning curve are the following.

Power law model

$$p = \alpha t^{-\beta} + \gamma + \varepsilon \tag{1}$$

where p is a general learning metric (for example, the fraction nonconforming of a manufacturing process), α is the fraction of nonconforming for the first learning cycle, β is the rate of learning, γ is the asymptotic fraction nonconforming and ε is the random error term ($\varepsilon = NID(0, \sigma^2)$).

Exponential model

$$p = \gamma + (p_0 - \gamma)\mathrm{e}^{-t/\tau} + \varepsilon \tag{2}$$

where p is a general learning metric (for example, the fraction nonconforming of a manufacturing process), p_0 is the initial fraction conforming, γ is the asymptotic fraction nonconforming, τ is the learning curve time constant and ε is the random error term ($\varepsilon = NID(0, \sigma^2)$).

The selection of a particular learning model is carried out on the basis of the specific application context (Muth, 1986; Schneiderman, 1988).

A recent paper by Zangwill and Kantor (1998) has introduced a kind of *unifying scheme* for the various models. The authors present five postulates that underlie certain types of industrial learning and give rise to a differential equation, which describes that learning. With this interpretation all models become parametric solutions of the Volterra-Lotka differential equation. This equation is used to describe the evolution of animal populations, according to the logic of the paradigm of predators and preys. In this context the preys are the fraction nonconforming, wastes and other inefficiencies that impair the operations of a process. The predators are management because they are attempting to eradicate the inefficiencies in order to improve the system (Zangwill & Kantor, 1998). The differential equation is:

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = -aW(t)S(t) \tag{3}$$

where $S(t)$ represents the number of preys, $W(t)$ is the number of predators and a is a scale factor.

With the aim of providing a 'preliminary estimate' of asymptotic performances of a manufacturing process, we build a learning model by means of information gathered during phase 1 (p-chart setting up) and phase 2 (verification of control limits). If $(\bar{t}_1, \bar{p}_1)$ and $(\bar{t}_2, \bar{p}_2)$ are, respectively, the coordinates of the average values of the fraction nonconforming related to the two phases, and $\bar{p}_i = (\alpha/\bar{t}_i) + \gamma + \varepsilon_i$ a simplified version of the power law model (a), with

$\beta = 1$, we may obtain a preliminary estimation of the learning process parameters from the following relationships:

$$a = \frac{\bar{p}_1 - \bar{p}_2}{1/\bar{t}_1 - 1/\bar{t}_2} \tag{4}$$

$$c = \bar{p}_1 - a/\bar{t}_1 \tag{5}$$

where $\bar{p}_1 = \Sigma_{i=1}^{k} p_i/n$ and $\bar{p}_2 = \Sigma_{i=1}^{m} p_i/m$ are the average values of fraction nonconforming, $\bar{t}_1$ and $\bar{t}_2$ are the average verification times; and k and m are the number of points analysed for each phase.

It can be shown that a and c are two unbiased estimators of α and γ: $E(a) = \alpha$; $E(c) = \gamma$.

As regards the variance, assuming statistically independent nonconforming fraction p_i, we can show that

$$\sigma_a^2 = \left(\frac{\bar{t}_2 \cdot \bar{t}_1}{\bar{t}_2 - \bar{t}_1}\right)^2 (\sigma_{\bar{p}_1}^2 + \sigma_{\bar{p}_2}^2); \qquad \sigma_c^2 = \left(\frac{\bar{t}_2}{\bar{t}_2 - \bar{t}_1}\right)^2 \sigma_{\bar{p}_2}^2 + \left(\frac{\bar{t}_1}{\bar{t}_2 - \bar{t}_1}\right)^2 \sigma_{\bar{p}_1}^2 \tag{6}$$

Now, define a percentage distance h from the asymptotic target, we can determine the time $t^\star$ to its achievement. Substituting this value in the learning model, we find

$$\gamma + \frac{h\gamma}{100} = \frac{\alpha}{t^\star} + \gamma \text{ and therefore } t^\star = \frac{\alpha}{h\gamma} \cdot 100 \tag{7}$$

As regards the variance of $t^\star$, we have

$$\sigma_{t^\star}^2 = \left(\frac{1}{h\gamma}\right)^2 \sigma_a^2 + \left(\frac{\alpha}{h\gamma^2}\right)^2 \sigma_c^2 \tag{8}$$

An example of application

Let us consider a process of frozen orange juice concentrate packing in 6-oz cardboard cans. The cans are formed on a machine by spinning them from a cardboard stock and attaching a metal bottom panel (Montgomery, 1996).

By inspection of a can, we are able to determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. We wish to set up a control chart to monitor the process and to improve the fraction of nonconforming cans.

To establish the control chart (phase 1), 30 samples of $n = 50$ cans each were analysed at hourly intervals over a three-shift period in which the machine was in continuous operation. Table 1 shows the data gathered.

When $\bar{p}_1 = \Sigma_{i=1}^{30} p_i/n = 0.2313$, a preliminary estimate of the upper and lower control limits of the fraction conforming control chart (p-chart) is the following:

$$UCL = \bar{p}_1 + 3\sqrt{[\bar{p}_1(1 - \bar{p}_1)/n]} = 0.2313 + 0.1789 = 0.4102$$

$$LCL = \bar{p}_1 - 3\sqrt{[\bar{p}_1(1 - \bar{p}_1)/n]} = 0.2313 - 0.1789 = 0.0524$$

where UCL is the upper control limit and LCL the lower control limit.

Table 1. *Fraction of nonconforming data collected in the process for 30 samples of* n = *50 cans (Montgomery, 1996)*

Sample No.	Number of non-conforming cans	p_i	Sample No.	Number of non-conforming cans	p_i
1	12	0.24	16	8	0.16
2	15	0.30	17	10	0.20
3	8	0.16	18	5	0.10
4	10	0.20	19	13	0.26
5	4	0.08	20	11	0.22
6	7	0.14	21	20	0.40
7	16	0.32	22	18	0.36
8	9	0.18	23	24	0.48
9	14	0.28	24	15	0.30
10	10	0.20	25	9	0.18
11	5	0.10	26	12	0.24
12	6	0.12	27	7	0.14
13	17	0.34	28	13	0.26
14	12	0.24	29	9	0.18
15	22	0.44	30	6	0.12

As Montgomery states, two points plot above the upper control limit (samples 15 and 23). The related assignable causes are detected and removed. Eliminating these points, the new revised control limits become:

$$\bar{p}_1 = \sum_{i=1}^{28} p_i/n = 0.2150$$

$UCL = 0.3893$ and $LCL = 0.0407$

Sample 21 exceeds the new upper control limit, however a further analysis of the data does not produce any reasonable assignable cause. We may conclude that the process is in control. The revised control limits may be adopted for monitoring current production.

We observe that the process nonconforming fraction is too high. A detailed analysis of the process indicates that several adjustments can be made on the machine. After these interventions an additional 24 samples are collected, with the aim of verifying the process improvement (phase 2). Table 2 shows the new gathered data.

Table 2. *Fraction of nonconforming data collected for additional 24 samples of* n = *50 cans (phase 2 control limits verification) (Montgomery, 1996)*

Sample No.	Number of non-conforming cans	p_i	Sample No.	Number of non-conforming cans	p_i
31	9	0.18	43	3	0.06
32	6	0.12	44	6	0.12
33	12	0.24	45	5	0.10
34	5	0.10	46	4	0.08
35	6	0.12	47	8	0.16
36	4	0.08	48	5	0.10
37	6	0.12	49	6	0.12
38	3	0.06	50	7	0.14
39	7	0.14	51	5	0.10
40	6	0.12	52	6	0.12
41	2	0.04	53	3	0.06
42	4	0.08	54	5	0.10

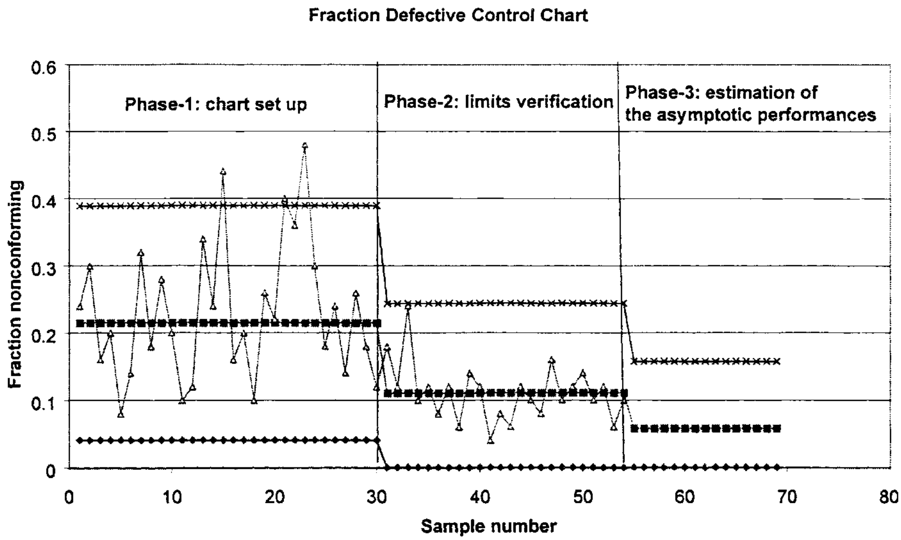

Figure 1. *Fraction nonconforming control chart for samples of* n = 50 *elements. The figure shows the control limits calculated in the chart setting-up phase (phase 1) and in the control limits verification (phase 2) (Montgomery, 1996). The third phase regards the estimation of the 'asymptotic' control limits determined by the process learning curve.*

As it appears from Fig. 1, we obtain a considerable reduction of nonconforming fraction. Montgomery (1996, pp. 258-259) said "It is not unusual to find that the process performance improves following the introduction of formal statistical process-control procedures, often because the operators are more aware of process quality and because the control chart provides a continuing visual display of process performance".

With these new data the process nonconforming fraction becomes

$$\bar{p}_2 = \sum_{i=31}^{54} p_i/n = 0.1108$$

The difference between the two average nonconforming fractions can be tested by means of the following hypothesis testing:

$$H_0: p_1 = p_2$$

$$H_1: p_1 > p_2$$

An approximate test based on the normal approximation to the binomial is:

$$Z_0 = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{[\bar{p}(1-\bar{p})\ (1/n_1 + 1/n_2)]}}; \text{ where } \bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2}$$

Substituting the values obtained, we find $Z_0 = 7.10 > Z_{0.05} = 1.645$. Consequently, the null hypothesis is rejected in favour of the alternative hypothesis (Montgomery, 1996).

Re-estimating the control limits for the nonconforming fraction, one obtains $UCL = 0.2240$ and $LCL = 0$ (see Fig. 1). Hypothesizing the learning model $\bar{p}_i = (\alpha/\bar{t}_i) + \gamma + \varepsilon_i$, we determine the asymptotic nonconforming fraction of the process, and the necessary time to achieve it.

Applying equations (4) and (5), we have:

$$a = \frac{\bar{p}_1 - \bar{p}_2}{1/\bar{t}_1 - 1/\bar{t}_2} = \frac{0.2150 - 0.1108}{1/15 - 1/42} = 2.43$$

$$c = \bar{p}_1 - a/\bar{t}_1 = 0.2150 - 2.43/15 = 0.053$$

where $\bar{t}_1$ and $\bar{t}_2$ are, respectively, the average times related to the two phases of p-chart setting up and control limits verification.

We assume for c and a a normal distribution. The c statistic is the estimation of the asymptotic fraction nonconforming of the process. It represents the fraction value that can be asymptotically achieved by the process as a consequence of the learning mechanism.

By the c value, we may determine the asymptotic control limits of the p-chart (see Figure 1):

$$UCL = c + 3\sqrt{[c(1 - c)/n]} = 0.053 + 0.095 = 0.148$$

$$LCL = c - 3\sqrt{[c(1 - c)/n]} = 0$$

The uncertainty associated with the estimation of a and c is (see equation (6)):

$$s_a = \sqrt{\left[\left(\frac{\bar{t}_2 \cdot \bar{t}_1}{\bar{t}_2 - \bar{t}_1}\right)^2 (s_{\bar{p}_1}^2 + s_{\bar{p}_2}^2)\right]} = 1.71$$

$$s_c = \sqrt{\left[\left(\frac{\bar{t}_2}{\bar{t}_2 - \bar{t}_1}\right)^2 s_{\bar{p}_2}^2 + \left(\frac{\bar{t}_1}{\bar{t}_2 - \bar{t}_1}\right)^2 s_{\bar{p}_1}^2\right]} = 0.076$$

where s_a and s_c are, respectively, the estimation of the standard deviation of a and c statistics, and $s_{\bar{p}_1} = \sqrt{[\bar{p}_1(1 - \bar{p}_1)/n]} = 0.058$, $s_{\bar{p}_2} = \sqrt{[\bar{p}_2(1 - \bar{p}_2)/n]} = 0.044$, the estimation of the standard deviation of $\bar{p}_1$ and $\bar{p}_2$.

The 95% two-sided confidence interval for the two parameters is:

$$\gamma = c \pm z_{0.025} s_c = 0.058 \pm 2 \cdot 0.076$$

$$\alpha = a \pm z_{0.025} s_a = 2.36 \pm 2 \cdot 1.71$$

As regards the time $t^\star$ to achieve a prefixed percentage distance from the asymptotic value, for example $h = 10\%$, we have

$$t^\star = \frac{\alpha}{h\gamma} \cdot 100 = \frac{2.43}{10 \cdot 0.053} \cdot 100 = 460 \text{ hours}$$

This value is about 19 days of continuous process operation. The standard deviation of $t^\star$ is $s_{t^\star} = 736$ hours.

Conclusions

The paper presents a method for the estimation of asymptotic performances of a manufacturing process. The method is based on the knowledge of the process learning curve and the information collected during the setting-up phases of a p-chart.

With a limited effort the method is able to give a preliminary estimation of the performance target reachable by the process and the time to achieve it.

The main novelties of the method are:

- the possibility to estimate the asymptotic variability of a process;
- the evaluation of the coherence between the asymptotic process nonconforming fraction and the related design specifications;
- the possibility to select among more alternative processes those more 'capable', from the asymptotic performances point of view;
- providing a simple approach for evaluating the period of revision of process control limits and their asymptotic values.

Further developments of the method are finalized to the definition of a procedure able to automatically adapt a new estimation to continuous information collected by the process.

References

ABERNATHY, W.J. & WAYNE, K. (1974) Limits of the learning curve, *Harvard Business Review*, September-October, pp. 109-118.

ADLER, P.S. & CLARK, K.B. (1991) Behind the learning curve: a sketch of the learning process, *Management Science*, 37, pp. 267-281.

BAILEY, C.D. & MCINTYRE, E.V. (1997) The relation between fit and prediction for alternative forms of learning curves and relearning curves, *IIE Transaction*, 29, pp. 487-495.

BOX, G.E.P. & LUCEÑO, A. (1997) *Statistical Control by Monitoring and Feedback Adjustment* (New York, John Wiley & Sons).

CHERRINGTON, J.E., LIPPERT, S. & TOWILL, D.R. (1987) The effect of prior experience on learning curve parameters, *International Journal of Production Research*, 25, pp. 399-411.

DADA, M. & MARCELLUS, R. (1994) Process control with learning, *Operations Research*, 42, pp. 323-336.

DUNCAN, A.J. (1986) *Quality Control and Industrial Statistics*, 5th Edn (Boston, MA, Irwin/McGraw-Hill).

FINE, C.H. (1986) Quality improvement and learning in productive systems, *Management Science*, 32, pp. 1301-1315.

FRANCESCHINI, F. & ROSSETTO, S. (1995) Quality & innovation: a conceptual model of their interaction, *Total Quality Management*, 6, pp. 221-229.

FRANCESCHINI, F. & ROSSETTO, S. (1998) On-line service quality control: the 'Qualitometro' method, *Quality Engineering*, 10, pp. 633-643.

KANTOR, P.B. & ZANGWILL, W.I. (1991) Theoretical foundation for a learning rate budget, *Management Science*, 37, pp. 315-330.

LI, G. & RAJAGOPALAN, S. (1998) Process improvement, quality, and learning effects, *Management Science*, 44, pp. 1517-1532.

LIPPERT, S. (1976) Accounting for prior practice in skill acquisition studies, *International Journal of Production Research*, 14, pp. 285-293.

MONTGOMERY, D.C. (1996) *Introduction to Statistical Quality Control*, 3rd Edn (New York, John Wiley & Sons).

MUKHERJEE, A.S., LAPRÉ, M.A. & VAN WASSENHOVE, L.N. (1998) Knowledge driven quality improvement, *Management Science*, 44, pp. 535-549.

MUTH, J.F. (1986) Search theory and the manufacturing progress function, *Management Science*, 32, pp. 948-962.

SCHNEIDERMAN, A.M. (1988) Setting quality goals, *Quality Progress*, 21, pp. 51-57.

VENEZIA, I. (1985) On the statistical origins of the learning curve, *European Journal of Operational Research*, 19, pp. 191-200.

ZANGWILL, W.I. & KANTOR, P.B. (1998) Toward a theory of continuous improvement and the learning curve, *Management Science*, 44, pp. 910-920.